Dimension of Non-Conformal Iterated Function Systems on the Plane

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Abstract

The topics studied in this thesis belong to the area of fractal geometry. This field is concerned with the study of objects with a structure that differs from classical euclidean geometry. Namely, these objects have a non-integer dimension and one of the main objectives of fractal geometry is to develop methods to estimate it.

In this master's thesis we study Non-Conformal Iterated Function systems (IFS). An IFS is a finite list of contractions. We analyse the fractal dimension of the attractors of an IFS, that is, the fractal dimension of some invariant set that arises after repeated applications of the maps in the IFS. The non-conformal IFSs are composed by maps which can contract with different ratios along different directions. Hence, compared to the more studied conformal case, where the maps in the IFS are contractions of the same ratio in every direction, the analysis of the dimension of the attractor becomes more involved.

In this thesis, we first introduce the main concepts and methods required for the study of the attractor of conformal and non-conformal IFSs. Then we study a seminal paper in the field of non-conformal IFSs [1] by Kenneth Falconer. There, the author presented a "tamed" version of a non-conformal IFS in which even though the maps are contractions with different ratios along different directions, these ratios are not allowed to differ arbitrarily. We point out a gap in the proof of [1, Lemma 4.2].

Finally, we study [2] and we present an extension of [2, Theorem 3] yielding a stronger result than the one in the original paper.

References

[1] Kenneth Falconer. "Bounded distortion and dimension for non-conformal repellers". In: *Mathematical Proceedings of the Cambridge Philosophical Society*. Vol. 115. 2. Cambridge University Press. 1994, pp. 315–334.

[2] Anthony Manning and Károly Simon. "Subadditive pressure for triangular maps". In: *Nonlinearity* 20.1 (2006), p. 133.