Abstract

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The history of graphical models independetly root from several scientific fields: from probability theory, from physics and from genetics as they provide an approach to deal with uncertainty.
"Graphical models are a marriage between probability theory and graph theory."
- Michael Jordan (1997)

The aim of this thesis is to show the relationship between graphs and joint distribution functions of random variables. Graphs provide us a convenient framework to intuitively represent the structure of the joint probability by some independence assumption (Pearl 1988, Lauritzen 1996, Whittaker 1990).

We introduce directed and undirected graphical models, in each case the main idea is to represent the random variables with the nodes of the graph and with edges represent relation between them, while missing edges represent some conditional independence between the two variables. A graph representation of the conditional independencies makes it simpler and computationally easier to find the family of the corresponding joint distribution and deriving its factorized form. The directed graphical models are known as *Bayesian Networks*, edges represent casual relations on a directed acyclic graph. Undirected edges represent a symmetric, conditional independence relation between variables and undirected models also called *Markov random fields.* The two models are different, however they are strongly related. We focus on the different properties of BNs and MRFs, and find graph related and probability theory related characteristics to equivalence. We also introduce decomposable and triangulated models, that are convenient to work with, and are especially adequate to use inference algorithms on them.

BNs and MRFs also give us different tools to compute probabilistic inference such as computing marginals of a distributions. For triangulated MRFs a junction tree of maximal cliques can be constructed in order to compute clique marginals.

We investigate the different types of generating classes of log-linear models, furthermore a log-linear model having graphical generating class if and only if it corresponds to an MRF, thus parameter estimation is simpler.

A graphical model can represent our domain and expert knowledge of the dependence structure of the variables. In the fourth chapter we show an example of building a DAG model from a real-life data to predict the outcome of a treatment of pain clinic patients.