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LIE GROUPS, LIE ALGEBRAS, REPRESENTATION THEORY, AND THEIR APPLICATIONS IN PHYSICS

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ABSTRACT

This abstract in your hands sums up the main concept of my thesis entitled *"Lie Groups, Lie Algebras, Representation Theory, and their Applications in Physics"*, which treats Lie groups, Lie algebras, and their representations.

My goals are twofold. First, I strive to introduce the basic notions of the theory of Lie groups in an elementary fashion. In particular, in the first two chapters, I have presented the basic notions of the theory of (matrix) Lie groups and their Lie algebras using only linear algebra, without requiring any knowledge of manifold theory. Second, I strive to provide some of the areas where Lie groups, Lie algebras, and their representations are mostly applied, namely Quantum Mechanics and Particle Physics.

As it is widely known, the importance of Lie groups, Lie algebras, and their unitary representations is stressed throughout, including not only the usual use of these to derive consequences for the theory of a "symmetry" generated by operators commuting with the Hamiltonian. Although Lie theory is widely used in both mathematics and physics, there is often a wide gulf between the presentations of the subject in the two disciplines: Physics books get down to business quickly but are often imprecise in definitions and statements of theorems, whereas mathematics books are more rigorous but often have a high barrier to entry. It is my hope that this thesis will somehow give a clear vision as well as an introductory bridge between both domains. In particular, the matrix approach allows for definitions that are precise but understandable, upon this, I do try to delve into some details of how Lie algebras and their representations are used in quantum mechanics and particle physics, I do include a discussion of the representations of $SU(3)$, which has obvious and wide applications to that field.