Abstract

Random walk on the Sierpiński graph and its oriented variations

Zsolt Bartis

2021

The main goal of this thesis is to dive into the analysis of random walks on complex structures, especially on the Sierpińksi graph. This was a popular area in Mathematics during the 20^{th} century with quite a lot of results in the literature.

The structure called "Sierpińksi gasket" was introduced in 1915 by a Polish mathematician, Wacław Sierpiński, and it is a fractal subset of $\mathbb{R}^2$. This gave inspiration and new directions to many mathematicians who started to analyze this structure and its variations like the "Sierpińksi lattice", which is the main objective of our thesis and we simply call it as Sierpińksi graph.

After giving a brief theoretical overview regarding the basic notions and definitions which are decisive for understanding the further research, we start to interpret some of the already existing results in the literature, from our own point of view. We create our own notational system for working with the Sierpiński graph. By coding its vertices and constructing a probability space, we managed to create a setting which helps us to make our calculations shorter and clearer. With the help of the method of generating functions we prove that the simple symmetric random walk on the non-oriented Sierpiński graph is subdiffusive. We also confirm this via computer simulation.

After having understood the behaviour of this random walk, we extend our model by orienting the graph. We can describe this extended model easily thanks to our notation. We present three possibilities for orientation, and we analyze the first one, namely, giving a random orientation independently to each elementary triangle.

It will turn out that this modification does not cause any significant change in the beaviour of the random walk. We confirm this with the help of our computer simulation. To have more proof, we also consider a non-randomized subcase, which gives the same result.