

Anastasiia Dzhioeva

Bachelor Thesis *Linear Algebra Methods in Extremal Set Systems*

Supervisor: Kiss Sándor

The main purpose of this project is to represent in practice that some basic linear algebra techniques can show their efficiency in proving bounds for extremal combinatorial problems. Linear algebra approaches help to solve many combinatorial problems, which could be very complicated to obtain using only the methods from the same mathematical area.

The main strategies for most of the problems which are represented use the possibility of working in a finite field. Mainly, we are introducing the characteristic vectors over this field, observing through the scalar product that they are linearly independent, and immediately concluding that the size of the linearly independent set of vectors is bounded from above by the dimension bound property. The dimension bound states that the size of a linearly independent set of vectors is at most the dimension of the space.

Similar approaches thought the notion of matrices as well as the polynomials are also represented in this project, where properties of both were used to obtain the result.

The various examples of the Odd/Even Town Theorems were considered to show the main idea of the thesis, as well as the different types of Ramsey numbers.

Some smaller but very helpful algebraic methods, such as properties and notions of the binomial coefficient, quadratic residues, and modulo, also showed their efficiency in the considered problems.