Abstract

Financial stress testing based on Kullback-Leibler divergence

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Stress testing is a technique used to test the resilience of institutions and investment portfolios against possible future financial situations. Traditional stress tests analyse subjectively chosen scenarios. A first attempt to overcome the subjectivity was made by Gerold Studer, who developed a traditional systematic stress test approach in his dissertation (1997) and a subsequent paper (1999). In this approach, a scenario realization is considered plausible if its Mahabolis distance from the expectation doesn't exceed a threshold and a loss function is minimized over the set of plausible scenarios. However, using Mahabolis ellipsoids as scenario sets is not always natural, and it surely doesn't work for credit risk models with discrete rating classes. Moreover, the risk factor distribution is assumed to be fixed ; hence, model risk is not taken into account in Studer's approach.

The thesis focuses on a paper of Thomas Breuer and Imre Csiszár that appeared in 2013. In this work, an alternative systematic way to perform stress tests is proposed. Mixed scenarios are used instead of pure ones, and the plausibility of a mixed scenario is measured by its Kullback-Leibler divergence with respect to some reference distribution.

The first chapter introduces important notions, including multivariate normal distribution, moment generating function, discrete and general versions of Kullback-Leibler divergence.

The second chapter states the Stein Lemma and proves it via the method of types that is a large deviation technique that considers sets consisting of sequences having the same empirical distribution. The Stein lemma provides a useful operational meaning of the Kullback-Leibler divergence: it is the optimal exponent of the type 2 error probability in the hypothesis test that tries to distinguish the distributions.

The third and final chapter presents the results of the Breuer-Csiszár paper mentioned above. The maximum loss theorem explicitly provides the worst-case distribution. As a first application of the maximum loss theorem, the worst-case distribution and loss are determined for linear portfolio loss with normal risk factor reference distribution. Probably due to its routine nature and relative lengthiness, the proof of the corresponding proposition is omitted in the Breuer-Csiszár paper. Based on the discussions with my supervisor, I proved it using the theory of moment generating functions. Moreover, the chapter describes two other applications from the Breuer-Csiszár paper along with supplementary numerical calculations.