Elephant Random Walk with General Step Distribution

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The aim of this thesis is to study and generalize the model of the so-called elephant random walk, which is a one-dimensional discrete random walk on the integers with memory. The elephant random walk model that we have generalized in this work start with a $+1$ or -1 step with $Rademacher(p)$ distribution, and henceforth every step of the walk is chosen uniformly from the previous steps multiplied by $+1$ or -1 with probability p and $1-p$. The asymptotic almost sure behaviour as well as the weak limit of this random walk has been examined extensively in the three regimes of the memory parameter, the diffusive-, the critical and the superdiffusive regime. Furthermore, in the diffusive regime, although the limiting distribution is not known, the first four moments of the almost sure limit has been determined.

Our generalization uses an arbitrary sequence of bounded, independent, identically distributed random variables $\xi_1, \xi_2, \dots$ to define the steps of the random walk. The first step of the walk is ξ_1, and every following step is either chosen uniformly from the previous steps with probability α or chosen from the the given i.i.d. sequence with probability $1-\alpha$. In this thesis we study the almost sure limiting behaviour of this random walk in two regimes of the memory parameter, during which we could verify that the existing almost sure convergence theorems hold in this generalized case. In the superdiffusive regime, we calculated the first four moments of the almost sure limit of the random walk, which is considerably more complicated in this generalized model.

Our approach to study the elephant random walk is based on martingales. We constructed a square-integrable martingale, that has bounded quadratic variation, which is the reason why we needed to define the step distribution bounded. A natural way to further generalization in the future

is to drop the boundedness condition on the step distribution. In this case one needs to search for an other method to study the elephant random walk, since the quadratic variation is not bounded.