Bachelor Thesis abstract

Sub-expansion complexity of singularities in dispersing billiards -
a computer assisted approach

Áron Millavári

2021

The objective is to prove that in dispersing billiard models the decay of correlation is exponential. This has been proven by Lai-Sang Young in 1998, but only in two dimensions. Thus, I wished to lay down some ideas for such a computer assisted proof that would be expandable to 3 dimensions. The essence of this new approach is to check the condition for the complexity of singularities, by using the Bálint-Tóth theorem. The theorem uses the size of expansion as one of its requirement, which increases by every none tangential collusion. The goal is to limit the number of tangential hits to two. Thus, the program creates all three to five long circle sequences, and checks their solvability if they have 3 grazing hits (for one specific pair of radii). The system of equations created for each individual sequence are not solved, they are tested for their Gröbner basis. Since the Gröbner basis can be used to decide whether a system can be solved or not.

The Bálint-Tóth theorem can be used for different radii, in 2 dimensions three radii were tested. The first is 9/25 and 3/20, this is one of the smallest possible choice, thus in this case the size of expansion increases sharply, and the theorem can be used. The second pair is 4/10 and 3/10 this is an interesting choice for a different reason, in this case four consecutive tangential line segments are possible, making the application of the theorem almost impossible. The third is 3/7 and 3/20, for this pair the theorem can be used as well.

As for three dimensions it is clear that a modified system would work, one of the main results of this thesis is that a well written system can decrease calculation time drastically, and finding the Gröbner basis of more but simpler systems of equations are way faster than finding the basis of one difficult one. Hence, a new approach would be necessary, a better system or something completely different. Similarly, the approach to generate the sequences is crucial, by a simple algorithm the number of sequences can be over 10 000 for all five long possibilities. Whether this method can be used in higher dimensions is yet to be seen.