1 Extract

In this B.Sc. thesis I aim to present a proof for the Cox-Durett shape theorem for First Passage Percolation (FPP). I wish to walk a reader with B.Sc. level Mathematics background through the path leading to the proof of the shape theorem. The purpose of this paper was to gain an understanding of the shape theorem, its proof and to learn about ergodic theory through the path leading to these. Initially First Passage Percolation was introduced as a model for fluid flow in porous medium by physicists. However, over the years it has become and abstract field of study for probability theorists. The Cox-Durett shape theorem states that in the model under reasonable conditions fluid-flow eventually assumes a limit shape.

Ergodic theory comes in the picture since FPP has a shift invariant property to it in distribution for which ergodic theory provides great results.

In Chapter 2 I present the model for FPP alongside basic definitions related to it.

In Chapter 3 I wish to walk through the fundamentals of ergodic theory. This chapter starts with defining stationary and ergodic measure preserving transforms and noting that this can be applicable in case of sequences as well. In the following section I state two different definitions for ergodicity and prove that they in fact are all the same. In the last two sections I aim to present a proof for Birkhoff's ergodic theorem, which interestingly yields Kolmogorov's Strong Law of Large Numbers in a special case, and for the Subadditive Ergodic theorem afterwards. In order to prove Birkhoff's ergodic theorem I utilize the Maximal ergodic lemma. In the process of proving the Subadditive Ergodic theorem I introduce Fekete's subadditive lemma, which is something alike to it in the non-random setting. In the following chapters I will use extensively the theorems and techniques developed in this one. While writing and understanding this chapter I relied mainly on the book *Probability: Theory and Examples* by Rick Durrett.

In Chapter 4 I discover the concept of time constant for FPP. I prove theorems that yield a good understanding of its nature and prove it exists under reasonable conditions. In order to grasp and present the contents in this chapter and the following one I relied on the survey article *50 years of first passage percolation* by Auffinger A. et al. This text however has many missing parts and gaping holes in its proofs. With the help of my supervisor, often by arduous work, we have managed to come up with fills for these holes in order to appreciate the proofs in their full splendor.

In the last Chapter I aim to put all the above mentioned pieces of knowledge together in order to present a proof for the Cox-Durett shape theorem. For this chapter I have run simulations of the model in order to illustrate its properties under certain assumptions.

In the Appendix one can find the program-code for the simulations I present in the last chapter. The code was written in Python language.