Densities of Minkowskian arrangements

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The famous Minkowski Theorem for lattice geometry gives a sharp upper bound for the volume of a origin-symmetric convex body which does not contain any non-zero point of a fixed lattice Γ in the d-dimensional Euclidean space $\mathbb{R}^d$ in terms of the area of the fundamental region of Γ. Inspired by this result, the notion of Minkowskian arrangement of convex bodies was first introduced by L. Fejes Tóth, who defined it as a family $\mathcal{F}$ of centrally symmetric convex bodies in the d-dimensional Euclidean space $\mathbb{R}^d$, with the property that no member of $\mathcal{F}$ contains the center of any other member of $\mathcal{F}$ in its interior. In particular, he showed that the Minkowski Theorem can be generalized to give sharp upper bound on the density of a Minkowskian arrangement of homothets of any convex body.

This thesis gives an overview of theorems concerning the maximal density of Minkowskian arrangements in various settings. Besides some well known results in the area, we prove generalizations of several theorems. We conclude by formulating questions and conjectures, providing preliminary observations related to them.