Thesis Abstract:

Moments and Cumulants in Renewal Processes.

Author: Ádám Bernát

Consultant: Péter Bálint

In this thesis we investigate a random walk derived from a renewal process whose arrival times have a polynomial tail distribution. The problem comes from the study of billiards and chaotic systems, where a special case of tail distribution (the case of $\beta = 2$, see below) was discovered to cause discrepancy between convergence in distribution and convergence of second moment. We extend this model to other polynomial tail distributions and study, analogously to the second moments, the behavior of higher moments and cumulants and their relation to convergence in distribution.

The two main objects investigates are the random walk derived from the renewal process, and an associated i.i.d. sum. After the mathematically correct introduction of the model we use the expansion of the characteristic function of the i.i.d. sum to describe its rate of convergence to the standard Gaussian. Here the fourth cumulant plays an important role

In the subsequent sections we consider the leading term that drives the growth of the fourth cumulant, as time tends to infinity. This quantity is studied using different interarrival distributions, first the exponential , then for the more general polynomial tails $\sim x^{-\beta}$ where $\beta \geq 4$.

We conclude by comparing the results on the random walk and on the i.i.d sum. Although not as clear indicators of anomaly as the results of the core phenomena about variances that inspired this thesis; our observations, especially in the $\beta = 4$ case, may provide a good starting point for further investigations on this topic.