Extremal problems of color-avoiding connectivity

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The concept of color-avoiding connectivity studies the resistance of colored networks. For example, given an edge-colored graph, we can check whether the graph remains connected if we remove the edges of a specific color. We call a graph edge-color-avoiding connected if with the removal of edges of any single color the graph remains connected, i.e., edge-color-avoiding connected graphs are completely resistant against color-specific attacks. We can also color the vertices of a graph instead of its edges, and consider a similar concept. In this case we can decide whether we want to remove those vertices as well, for which we are checking if they are color-avoiding connected, when we are removing vertices of their colors. This yields the definition of vertex-color-avoiding connectivity and internally vertex-color-avoiding connectivity.

We considered two extremal problems of color-avoiding connectivity, which naturally arise for each version of color-avoiding connectivity (edge, vertex and internally vertex). Namely, we determined the minimum number of edges from which a color-avoiding connected graph can be constructed on a given number of vertices with a given number of colors, and we investigated the problem of determining the maximum number edges that can be removed such that the graph remains color-avoiding connected. In real-life scenarios, both types of problems naturally occur: in the first one we want to minimize the construction costs, in the second one we want to minimize the maintenance costs.

We prove some interesting results regarding both topics. For the first problem we showed how many edges are needed to construct an edge-, vertex- or internally vertex-color-avoiding connected graph. For the second problem we proved that deciding whether there exists an edge-, vertex or internally vertex-color-avoiding connected spanning subgraph with a given number of edges is NP-complete. However, we gave polynomial time approximation algorithms for finding an edge-, vertex- or internally vertex-color-avoiding connected spanning subgraph with minimum number of edges.