Stability of Autonomous systems
Summary

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This thesis investigates the stability of autonomous systems, with a particular focus on practical applications of linear systems of ordinary differential equations, such as the Prey-Predator model. By means of rigorous theoretical examination and concrete illustrations, we investigated the importance of stability in understanding the operation of these systems.
Firstly, in Chapter 2, we defined the necessary theoretical groundwork to understand the properties of these systems. In the next chapter, Chapter 3, the influence of several factors, particularly eigenvalues and algebraic methods, on the stability of linear systems of differential equations is investigated. There, we analysed how the system behaves when the eigenvalues are different real numbers, repeated real numbers, and conjugated complex numbers. By applying the necessary mathematical manipulations, we were able to determine how the stability of a 2×2 system depends on the determinant and the trace of the Jacobian matrix. We obtained that for the equilibrium point to be stable or asymptotically stable, we need the determinant to be negative and the trace to be less than or equal to zero. A summary of the findings is provided in Proposition 3.2.1.
Afterwards, in Chapter 4, we apply the principles investigated previously to the real-life application, Prey-predator model with a constant quota of harvesting. An initial step involves the determination of the Jacobian matrices that correspond to possible positive equilibrium points. After determining the equilibrium points, the bifurcation point is calculated. Following that, we examined various scenarios, focusing on situations in which the harvesting rate is both equal to and less than the bifurcation point. To analyse stability in those specific cases, we computed the trace and determinant of the Jacobian matrix corresponding to the equilibrium point. Our investigation is complete and the results are summarized in a table.