Summary

Card shuffling Precision: Best Guessing Strategy and Convergence

In this thesis we investigate the aspects of card shuffling from a mathematical point of view. Such as, what the most likely order of the cards is after k shuffles or how far away a distribution is from uniform. Uniform distribution here is the ideal situation of a perfectly shuffled deck. With an in-depth analysis of the theoretical model and additional simulations, it is our aim to get a better understanding of the behaviour of the cards.

Firstly, we investigate the Gilbert-Shannon-Reeds model (GSR) for the riffle shuffle. By doing so, we are able to provide a best guess strategy and its corresponding probability of being correct after one riffle shuffle. With the help of computer computations we are able to extend the theory to multiple shuffles as well.

Secondly, we turn our attention to the concept of convergence to uniform distribution. As mentioned before, in an ideally shuffled deck, every possible order of the cards should be equally likely. According to the results proven by Bayer and Diaconis, approximately $2log_2(4n/3)$ shuffles are necessary for a deck of size n. And after this many shuffles we have a distribution close enough to uniform: a clear guideline for effective shuffling in practice.

Furthermore, we go deeper into the total variation distance between the card distribution and the uniform distribution. Since, total variation is a concept hard to measure, we make use of computer simulations. Every one of these simulations will be riffle shuffled according to the GSR model. The degree of variation among these simulations is directly related to the total variation distance. Although the simulations offer more insights, the total variation distance does not converge to zero but instead to a specific expected value, if the number of simulations is not high enough. We calculate this expected value and establish both lower and upper bounds for the total variation distance. This is done in different settings, first a more simplified reality where we find asymptotic expectations and bounds. And afterwards, a more realistic non-asymptotic expected value as well as non-asymptotic bounds on the total variation distance.

All in all, we get a better insight on the workings of the riffle shuffle and its effectiveness of randomizing a card deck in practice.