Abstract:

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For a given class of groups, it is an interesting question which groups are the closest to the class, that is, all of their proper subgroups are in the class but themselves are not. In this sense minimal non-cyclic, minimal non-Abelian, minimal non-nilpotent, minimal non-Iwasawa and minimal non-supersolvable groups have been examined. A group is called modular if its lattice of subgroups is modular. These groups are between Abelian and supersolvable groups, and the structure of their finite elements were described by Iwasawa in 1941.

The main goal of this thesis is to examine minimal non-modular groups. In the first chapter, we review the basic concepts of lattice theory, introduce modular lattices and present their well-known properties, as well as some useful statements about modular elements of arbitrary lattices. In the second chapter, we examine modular groups and provide a sketch of the proof of the theorem of Iwasawa, based on the book Subgroup Lattices of Groups by Roland Schmidt. In section 2.3, we prove a few new necessary or sufficient conditions for modularity of groups that we need later.

The main results of the thesis are in the third chapter, Minimal non-modular groups, in which we prove the following theorems.

1. If a group G is minimal non-modular then the order of G has at most 3 distinct prime divisors.
2. Suppose the order of G has three distinct prime divisors p, q, r. Then G is minimal non-modular if and only if $G/Z(G)$ is a non-modular group of order pqr, and every Sylow subgroup of G is cyclic.
3. Let G be a minimal non-modular group, and let the prime divisors of the order of G be $p > q$. If $P \in \mathrm{Syl}_p(G)$ and $Q \in \mathrm{Syl}_q(G)$, then exactly one of P and Q is a normal subgroup, and at least one is cyclic.