Zarankiewicz Numbers and Bipartite Ramsey Numbers

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Abstract

This thesis project enhances the computation of upper bounds for Zarankiewicz numbers through the development of an advanced Integer Linear Programming (ILP) model using the GLPK solver. The project begins by describing the foundational ideas and key bounds from previous studies. Building on these foundations, this thesis generalizes these bounds in a more comprehensive manner. Unlike previous works that focused on specific cases, such as small degrees in Collins et al.'s idea and maximum degrees in Dámasdi, Héger, and Szőnyi's approach, this project employs more sharped constraints in recursion to achieve potentially better bounds. Furthermore, this project explores the relationship between Zarankiewicz numbers and bipartite Ramsey numbers, demonstrating how the Zarankiewicz numbers can help in establishing the upper bounds for the bipartite Ramsey numbers. The main contribution of this work is to improve the results of upper bounds for Zarankiewicz numbers by developing an advanced integer linear programming (ILP) model using the GLPK solver, which generalizes the bounds, making the process more flexible and efficient, surpassing previous methods like those of Irving's and Roman's. This thesis contributes significantly to extremal graph theory by providing a robust computational tool and advancing the theoretical understanding of these bipartite graphs.