

Abstract : Ekeland's Variational Principle and the Mountain Pass Theorem.

Author : So Murata, Supervisor : Dr. József Kolumbán.

This thesis presents concise proofs of both Ekeland's variational principle and the mountain pass theorem, along with their applications. Historically, Ekeland's variational principle first appeared in his 1974 paper, "On the Variational Principle." This theorem can be used to find approximate minima of functions when the Bolzano–Weierstrass theorem is not applicable. The mountain pass theorem guarantees the existence of critical values. We provide two versions of the mountain pass theorem: the first from a differential geometric perspective and the second using Ekeland's variational principle. To maintain the conciseness of the paper, numerous propositions are included with proofs. We begin with topological preliminaries and gradually move into the proofs of the two main theorems, providing necessary tools from relevant subjects. Lastly, we consider an important corollary of Ekeland's variational principle and an application to PDE problems in Sobolev spaces.