The $5x + 1$ problem

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This thesis is about the $5x + 1$ problem, which spans a similar problem to the infamous Collatz Conjecture. The thesis analyses the $3x + 1$ *Collatz Function* first, then uses the definitions and intuition introduced on the next chapters about the $5x + 1$ function and conjecture, and its generalization $5x + d$.

The $5x + 1$ problem differs from the $7x + 1$ problem and others alike, since most of its trajectories grow slowest of similar problems, but still grows on average. It generates 3 "small" cycles spanning a huge portion of the lower numbers while spanning less and less of the larger numbers by virtue of increasingly rare convergent trajectories appearing, with the cycle ranges pairwise divergent. Lastly, there are infinitely many distinct divergent trajectories and all trajectories, including the convergent trajectories, have at least one element from an arithmetic progression of step-size coprime to 2 and 5, in their respective orbits, making the progression a valid witness to the conjectures stated. These facts are made clear on this thesis, marking my small contribution to demystifying this seemingly dead-end of Mathematics.