Bounding eigenvalues in Spectral graph theory

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Bachelor Thesis Summary

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In spectral graph theory, two well-known classical inequalities are Hong's inequality, which provides an upper bound for the spectral radius of a graph, and Wilf's inequality, which gives a lower bound for the clique number of a graph. These are expressed as $\lambda_1 \leq \sqrt{2m - n + 1}$ and $\frac{n}{n-\lambda_1} \leq \omega$, respectively, where n is the number of vertices, m is the number of edges, ω is the clique number, and λ_1 is the spectral radius (or largest eigenvalue) of the graph.

We demonstrate how the Motzkin and Straus theorem can be applied to prove the lower bound for the clique number, providing a full and detailed explanation of this theorem. Additionally, we will prove Hong's inequality.

We explore potential extensions of these inequalities. Let s^+ denote the sum of the squares of the positive eigenvalues of a graph. It is conjectured that $n-1 \leq s^+ \leq 2m-n+1$ for connected graphs. This conjecture can be proven for hyper-energetic and regular graphs. Another conjecture is that the lower bound for the clique number is $\frac{n}{n-\sqrt{s^+}}$. This has been proven for triangle-free graphs, weakly perfect graphs, and Kneser graphs by other authors. In this thesis, we additionally proved it for Paley graphs and wheel graphs.