Abstract

Consider the Navier-Stokes equations, which are partial differential equations that describe the flow of an incompressible viscous fluid under an external force. In this thesis, we study the sufficient conditions for the existence and uniqueness of a solution to the weak formulation of the problem in an open, bounded domain. We introduce the required tools to define and approach the problem, including Sobolev inequalities and extension operators. Next, we explain our results in the homogeneous case, when the boundary condition is 0. Afterwards, we work on the inhomogeneous case and introduce a method to deduce its weak solution from the homogeneous case. In both cases, uniqueness depends on what we call *the small data condition*. We also present an iterative method based on Banach's Fixed Point Theorem to find the weak solution of the equations.

To produce this work, we have thoroughly reviewed various sources from the literature on these topics to create a comprehensive description of the Navier-Stokes problem and its weak solution. This has allowed us to shed light on details that have usually been dismissed. Specifically, a lot of emphasis has been put on finding bounds for the constants involved in the small data condition, which are typically only referred to as *some constants that depend on the dimension and the domain*. In our work, we give precise values to these constants, so that we can immediately determine the uniqueness of a weak solution through the data of the problem (i.e. L^2 norm of the external force, viscosity of the fluid and diameter of the domain).