Overlapping self-affine carpets with the weak separation condition

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The study of self-affine carpets is a main highway in the survey of self-affine sets since, despite their simpler structure compared to general self-affine sets, they already show many key and generic features, in particular, they provide one of the simplest examples of sets with differing box-counting and Hausdorff dimensions.

Assuming separation among the cylinders (images of the attractor by the maps of the IFS), many aspects have been studied. Moreover, there have been papers assuming less, namely, the exponential separation condition. We studied with the constraint that the projection of the IFS to the x and y-axes satisfies the weak separation condition. This assumption is natural, since these projections are self-similar IFSs.

In the thesis, we computed the box-counting dimension in the above generality, and the Hausdorff dimension under the additional assumption that the system also resembles the Lalley-Gatzouras construction. This further assumption is rather technical, since it is assumed only because, in full generality, with only assuming separated cylinders, the Hausdorff dimension is unknown.

We proved that the Hausdorff dimension is the limit of the Lalley-Gatzouras formula taken on the n-fold compositions of the functions in the IFSs. For the box-counting dimension, we proved its existence and that it equals to the limit of the Feng-Wang formula taken on the n-fold compositions of the functions in the IFSs. These compare nicely to the self-similar case, where the Hausdorff (and box-counting) dimension equals the limit of the similarity dimension taken over the n-fold compositions of the functions in the IFSs.

We further showed in two examples that our theory can be practical by computing the dimensions of self-affine sets with previously unknown dimensions.