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Conditioning Branching Processes to Extinction

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Abstract

This thesis studies a question about branching processes: can every subcritical branching process be obtained by taking a supercritical one and conditioning it to die out?

Branching processes are models for populations where each individual produces a random number of offspring. A process is called subcritical if the population tends to die out, and supercritical if the population tends to grow. Conditioning a supercritical process on extinction means that we focus only on those cases where, despite the tendency to grow, the population eventually dies out.

We first consider the single-type case, where we construct a new generating function from a given subcritical one. This new function defines a supercritical process which, when conditioned on extinction, gives back the original subcritical process. We prove that such a construction always works when a fixed point greater than one exists, and we show that this fixed point is unique.

The main part of the thesis extends this idea to multitype branching processes. We prove that, under suitable conditions (subcriticality, continuity, positive regularity, and non-singularity), the generating function of a multitype process has a fixed point in $(1, \infty)^d$. Using a change of coordinates and Brouwer's Fixed Point Theorem, we show that this fixed point defines a supercritical process whose extinction-conditioned version has the same law as the original subcritical one.

We also show that, unlike in the single-type case, the fixed point may not be unique in the multitype setting. This means that more than one supercritical process can correspond to the same subcritical process after conditioning.

Finally, we explain how these results are useful in biological models. For example, a tumor that seems to shrink might actually come from a growing process that has been conditioned to die out. This makes the mathematical idea of extinction conditioning relevant to understanding real-world systems.