

FINAL EXAM QUESTIONS

- (1) **Brownian motion:** Martingales derived from B.M., properties of B.M., reflection principle, quadratic variation of B.M.
- (2) **Itô integral:** def. of Itô int. (w.r.t. Brownian motion), Itô isometry. Case of deterministic integrand (Gaussian process), Quadratic variation of Itô integral.
- (3) **Itô formula:** Itô process, Itô formula, stochastic integral w.r.t. Itô process, Itô formula for Itô processes, stochastic integration by parts, time-dependent Itô formula, multivariate Itô formula.
- (4) **Stochastic differential equations (SDE):** Stochastic exponential and stochastic log. Solution of general linear SDE. Girsanov's theorem.
- (5) **Famous Itô diffusion processes and their properties:** O-U process, Geometric Brownian motion, Brownian bridge, Bessel process, Bessel squared process, C.I.R. process, stochastic logistic equation.
- (6) **SDEs and PDEs:** Diffusions and related elliptic PDEs (Poisson, Helmholtz). Diffusions and related parabolic PDEs (Kolmogorov forward/backward, Feynman-Kac formula). Stationary distribution of 1-dimensional diffusion process.