Convex Geometry (BMETE94AM18)

Topics for the final exam

Affine geometry and convex combinations

Affine and convex sets, affine dependence and independence, affine and convex combinations, affine hull and its properties, convex hull and its properties

Theorems of Radon, Carathéodory and Helly

Theorems of Radon, Carathéodory and Helly, their applications and variants: colorful Carathéodory theorem, Jung's theorem, etc. Linear functionals, their connection with scalar product and hyperplanes.

Minkowski sum, separation

Minkowski addition and its properties. Support function. Isolation and separation, isolation theorem for open convex and arbitrary convex sets, separation theorems.

Extremal and exposed points

Extremal and exposed points, connection between extremal points and linear functionals, Krein-Milman Theorem. Straszewicz Theorem. Faces of convex sets.

Indicator function of convex bodies, Euler characteristic

Indicator function of closed, convex sets and its properties. Algebras of closed/compact convex sets, valuations, Euler characteristic and its properties. Convex polytopes, polyhedral sets and their connections. Face structure of polytopes. The *f*-vector of polytopes. Euler's theorem.

Polarity

Polarity and its properties. Polar of a polytope. Dual polytope and its properties.

Hausdorff distance and its properties

Family of convex bodies as a metric space. Hausdorff distance of sets. Cauchy sequences, Blaschke's selection theorem, approximation by polytopes