Partial Differential Equations 1

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1. **Basics.** Definition of a PDE, well-posedness. Second-order linear PDEs, their classification, examples: Poisson equation, heat equation, wave equation.

2. **Distributions.** The set $\mathcal{D}(\Omega)$, definition of a distribution (using the sequential continuous and the boundedness properties, their connection). Regular distribution, Dirac-delta. Derivative of a distribution, the derivative of a regular distribution defined on $\mathcal{D}(\mathbb{R})$.

3. **Fundamental solutions.** Cartesian product of distributions. Convolution of distributions. Definition of a fundamental solution, its connection to the solution of a linear PDE.

4. **Time dependent equations.** Classical solution of the wave and the heat equations (defined on $(t, x) \in \mathbb{R}_0^+ \times \mathbb{R}^n$). General (weak) solutions of these equations (in the distributional sense). The connection between the classical and the weak solutions. Comparison: number of classical solutions, speed of the wave/heat propagation.

5. **Boundary-value problems (BVPs).** Classical first, second, and third-type BVPs. Theorem about the number of solutions. Eigenvalue problems (first, second, third type). Theorem about operator L and its eigenvalues and eigenfunctions. The method of Fourier.

6. **Weak solutions of BVPs.** Definitions of $H^k(\Omega)$ and $H_0^k(\Omega)$ (completion, general derivatives, their connections). Weak form of BVPs. Connection between the classical and weak forms of BVPs. Theorem about the solution of a weak BVP, connection to the classical BVP.