**BMETE95MM41**, **Stochastic Analysis** (8 credit course)

**FINAL EXAM QUESTIONS**

(1) **Brownian motion:** Paul Lévy's construction of B.M., Martingales derived from B.M., properties of B.M., B.M. is nowhere differentiable, strong Markov property, reflection principle, quadratic variation of B.M.

(2) **Itô integral:** def. of Itô int. (w.r.t. Brownian motion), Itô isometry. Difference between Stieltjes and Itô integrals. Case of deterministic integrand (Gaussian process), martingale property of Itô int. Quadratic variation of Itô integral.

(3) **Itô formula:** Itô process, Itô formula, stochastic integral w.r.t. Itô process, Itô formula for Itô processes, stochastic integration by parts, time-dependent Itô formula, multivariate Itô formula, discrete Itô formula for simple random walk.

(4) **Martingales in Stoch. Anal.:** Harmonic functions and martingales. An example of a process with vanishing drift which is not a martingale. Paul Lévy's characterization of B.M. Martingale representation theorem. Change of measure: Girsanov's formula.

(5) **Stochastic differential equations (SDE):** Existence and uniqueness of strong solution of SDE. Counterexamples: what can go wrong if the Lipschitz condition fails. Stochastic exponential and stochastic log. Solution of general linear SDE.

(6) **Famous Itô diffusion processes:** Strong vs weak solution of SDE, famous diffusion processes and their properties: O-U process, Geometric Brownian motion, Brownian bridge, Bessel process, Bessel squared process, C.I.R. process, stochastic logistic equation, Tanaka's SDE.

(7) **SDEs and PDEs:** Diffusions and related elliptic PDEs (Poisson, Helmholtz). Infinitesimal generator, Dynkin's formula. Diffusions and related parabolic PDEs (heat equation, Kolmogorov forward/backward, Feynman-Kac formula). Stationary distribution of 1-dimensional diffusion process. Recurrence vs positive recurrence of diffusion processes. Black-Scholes model, B.-S. PDE, B.-S. formula.