

Stochastic Analysis with Applications, BMETE95MM04

(5 credit course, last taught in 2018, replaced with new courses in 2021)

FINAL EXAM QUESTIONS

1. **Introduction, reiteration:** conditional expectation, martingale, stopping time, optional stopping theorem, theorems about discrete-time (sub)martingales: Doob-Meyer-decomposition, submartingale inequality, applications
2. **Brownian motion (BM):** definition, Lévy's construction of BM, martingales related to BM, quadratic variation of BM, reflection principle and applications, symmetries of BM (scale invariance, time reversal, time inversion)
3. **Stochastic Analysis for simple random walk (SRW):** martingales related to SRW, discrete stochastic integral, discrete Ito's formula, Tanaka's formula for SRW, martingale representation theorem for SRW
4. **Stochastic Analysis for Brownian motion A:** Ito integral, Ito isometry, comparison of Ito and Stieltjes integrals
5. **Stochastic Analysis for Brownian motion B:** the stochastic integral w.r.t. Brownian motion as a stochastic process, its quadratic variation, Ito's formula (simplest case), comparison of chain rule of classical calculus and Ito's formula, Ito integral w.r.t. BM in the case when the integrand is deterministic
6. **Stochastic Analysis for Brownian motion C:** Ito processes, stochastic integral with respect to an Ito process, Ito formula for Ito processes, integration by parts, multi-dimensional case, applications, Lévy's characterization of BM
7. **Introduction to stochastic differential equations (SDE):** Ito diffusion process, solution of general linear SDE, examples: geometric Brownian motion, Ornstein-Uhlenbeck process, Brownian bridge, Bessel process