Exercises

(1) Compute the volume element of a torus with different parametrizations.
(2) Compute the volume element of the sphere with different parametrizations.
(3) Compute the volume element of the ellipsoid with different parametrizations.
(4) Integrate the distance from a plane on a sphere, ellipsoid, torus.
(5) Compute $d\alpha$ if α is equal to
   (a) $xydx + x^2zdy + yz^3dz$,
   (b) $xy^2dx + x^2yzdy + 2xyz^3dz$,
   (c) $2x^3z^2dx + x^2ydy + 2xz^3dz$,
   (d) $x^2ydx \wedge dy + 4yz^2dy \wedge dz + xyzdx \wedge dz$,
   (e) $2x^3z^2v^2dx \wedge dv + x^2ydy \wedge dx + 2xz^3dz \wedge dv$,
   (f) $x^2yvdx \wedge dy \wedge dv + 4yz^2v^2dy \wedge dz \wedge dv + xyzdx \wedge dz \wedge dy$.
(6) Is there any β such that $d\beta = \alpha$ in the previous exercise?
(7) Compute $\int_M \omega$ if M is the straight line segment connecting $(1, 2)$ and $(-3, -3)$ and ω is the form $xydx + x^2zdy + yz^3dz$.
(8) Compute $\int_M \omega$ for all the 1-forms in exercise (5).