FUNCTIONAL ANALYSIS I

1. Metric spaces. Compact set, sequentially compact set, totally bounded set. Relative compact set. Arzela-Ascoli theorem.
2. Normed spaces, Banach spaces. Series in normed spaces. Finite dimensional normed spaces.
3. Linear operators on normed spaces. Boundedness and continuity. The normed space of bounded linear operators.
4. Baire's theorem. The principle of uniform boundedness. Banach-Steinhaus theorems.
5. The open mapping theorem, Banach's theorem on the bounded inverse, closed graph theorem.
6. Linear functionals and dual spaces. Hahn-Banach extension theorems, Hahn-Banach separation theorems. Reflexivity.
7. Weak convergence, weak *-convergence. Banach-Alaoglu theorem, Eberlein-Smulian theorem.
8. Compact operators, Schauder's theorem, the Fredholm alternative.
9. The spectrum of a bounded linear operator. Neumann series, the resolvent. The spectral radius formula. The spectrum of a compact operator.
10. Inner product spaces. Projection theorem. Orthonormal bases. Riesz-Fischer theorem.
11. Riesz's representation theorem, the Hilbert space adjoint of a bounded linear operator.
12. The spectral theorem for normal compact operators.