CEU *Probability 2*: List of theorems for the exam

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1. Lévy's construction of 1-dimensional Brownian motion. For any $\epsilon > 0$ it is almost surely $(1/2 - \epsilon)$-Hölder continuous, but nowhere differentiable.

2. Strong Markov property, reflection principle, $M_t = |B_t|$ in distribution for fixed t, and statement of Lévy's theorem that $M_t - B_t = |B_t|$ in distribution as processes.

3. The zero set of 1-dim BM is almost surely a closed set without isolated points, of Hausdorff-dimension $1/2$ (assuming the theorems in the previous items).

4. Almost sure uniqueness of the maximum place of B_t in $[0, 1]$, and the ArcSine Law for its location.

5. Skorokhod's embedding for the special case of a two-valued random variable. Statement and proof of Donsker's invariance principle, assuming Skorokhod's embedding theorem.

6. Definitions of Lévy processes, infinitely divisible distributions, α-stable Lévy processes. Proof that the normalized sum of iid Cauchy's is Cauchy.

7. Infinitesimal generator for continuous time Markov chains and for BM. The discrete Laplacian for discrete time Markov chains. Stationary and reversible measures.

8. Convergence to stationarity in total variation distance for finite irreducible aperiodic Markov chains.

9. The total variation mixing time, with cutoff, for lazy RW on the hypercube, via coupon collecting.

10. Pólya's theorem for recurrence vs transience on $\mathbb{Z}^d$, using the Azuma-Hoeffding large deviation inequality.

11. The spectral radius (exponential rate of decay for the return probability) on the d-regular tree is $2\sqrt{d-1}/d$. The notion of amenability. The easy direction of the Kesten-Cheeger theorem: a Følner sequence implies that the Markov operator has norm 1.

12. The spectrum of finite reversible Markov chains. The bound implied by an absolute spectral gap on the uniform distance from stationarity. The notion of expanders.

13. Ergodicity and mixing of probability measure preserving transformations. Birkhoff's ergodic theorem.