Basc probability - additional exercises

1. Suppose we are told that, on average, there are two clicks per minute on a specific web page. We start to observe this web page and decide to model the waiting time till the first click Y (in minutes) using an exponential distribution. What is the probability that we have to wait at most 40 seconds to observe the first click? If there is no click in the first minute, what is the probability that we have to wait for another 40 seconds for the first click?

2. The distributions of random variables X and Y are exponential with parameters 2 and 3 respectively. What is the distribution of $\min\{X, Y\}$?

3. Let $X \sim Exp(\frac{1}{2})$. $P(X > 1) = ?$, $P(X > 3 | X > 2) = ?$

4. Assume that we have an unfair (biased) die that is labeled with the numbers $0, 1, 2, 3, 4, 5$ and that gives these values with probabilities $0.35, 0.2, 0.05, 0.05, 0.1, 0.35$, respectively. We roll this die twice. Let X_1 and X_2 denote the result of the first and second rolls. What is the distribution of $Y = X_1 + X_2$?

5. (Second homework, deadline: 5th of September, 12:15), you should bring it to the lecture) We flip three ten-forint coins. Let X denote the number of heads we get. Then we flip as many twenty-forint coins as many heads we get with the ten-forint coins, i.e., we flip X many twenty-forint coins. Let Y denote the number of twenty-forint heads we get. Specify the joint distribution of X and Y! Calculate the marginal distribution of Y!