

NAME: NEPTUN-CODE: MAJOR:

Probability Theory 1 2nd midterm, 23rd November 2023.

8:10–8:55

*Working time: 45 minutes. Only a simple scientific, non-programmable calculator can be used.
Maximum score (with the bonus exercise): 24 points, but we consider 20 points already as 100%.*

1. In a school with 400 students, the canteen serves only two types of dishes, pottage and stew. Each student chooses the stew with probability 0.8 and the pottage with probability 0.2. The choices of the students are independent of each other. The chef decides to cook $320 + n$ many stews and $80 + n$ many pottages. How shall he choose the value of n in order to be able to serve every student's choice with probability 0.97? (7 points)
2. Let ξ be a random variable with exponential distribution of parameter $\lambda = 1$, and let $X = \sqrt{\xi}$.
 - (a) Find the probability density function of X ! (4 points)
 - (b) Calculate the conditional probability $\mathbb{P}(X > 2|X > 1)$! (3 points)Extra $\mathbb{E}(X)=?$ (4 points)
3. Let (X, Y) be a pair of random variables jointly uniformly distributed on the trapezoid formed by the vertices $(0, 0)$, $(1, 0)$, $(1, 1)$, $(0, 2)$.
 - (a) What is the probability that $Y > X$? (3 points)
 - (b) What is the marginal density of X ? (3 points)

