

GROUP A

① RS: $X \sim \text{EXP}\left(\frac{1}{70}\right)$

CU: $Y \sim \text{EXP}\left(\frac{1}{65}\right)$

a) $P(\text{LOSER THROWS} \geq 60) = P(\text{BOTH} \geq 60) =$

$P(X \geq 60, Y \geq 60) \stackrel{\text{INDEP.}}{=} P(X \geq 60) \cdot P(Y \geq 60) = e^{-\frac{60}{70}} \cdot e^{-\frac{60}{65}} = 0.1677$

b) $f(x, y) = f_X(x) \cdot f_Y(y) = \frac{1}{70} \cdot e^{-x/70} \cdot \frac{1}{65} \cdot e^{-y/65}$

c) $P(\text{RS WINS}) = P(X \geq Y) = \int_{x>y} \int f(x, y) dx dy$

$= \int_0^{\infty} \int_y^{\infty} \frac{1}{70} \cdot e^{-x/70} \cdot \frac{1}{65} \cdot e^{-y/65} dx dy =$

$= \int_0^{\infty} \frac{1}{65} \cdot e^{-y/65} \cdot e^{-y/70} dy = \frac{1/65}{1/65 + 1/70} = 0.5185$

BONUS: $E(\sqrt{X}) = \int_0^{\infty} \lambda \cdot e^{-\lambda x} \cdot \sqrt{x} dx =$

$\int_0^{\infty} \lambda \cdot e^{-\lambda \cdot y^2} \cdot 2y^2 dy = \text{Ü}$

LET $\lambda = \frac{1}{2} \cdot \frac{1}{\sigma^2}$

$y = \sqrt{x}$
 $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$
 $2\sqrt{x} dy = dx$

THUS $\sigma = \frac{1}{\sqrt{2\lambda}}$

$$\begin{aligned} \textcircled{1} &= 2\lambda \cdot \int_0^{\infty} e^{-y^2/2\sigma^2} \cdot y^2 dy = \lambda \cdot \int_{-\infty}^{\infty} e^{-y^2/2\sigma^2} y^2 dy \\ &= \lambda \cdot \sqrt{2\pi} \cdot \sigma \cdot \underbrace{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\sigma} \cdot e^{-y^2/2\sigma^2} dy}_{=1} = \end{aligned}$$

VARIANCE OF $\mathcal{N}(0, \sigma^2) \rightarrow \sigma^2$

$$= \lambda \cdot \sqrt{2\pi} \cdot \sigma^3 = \lambda \cdot \sqrt{2\pi} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \cdot \lambda^{-\frac{3}{2}} = \frac{\sqrt{\pi}}{2 \cdot \sqrt{\lambda}}$$

$$\textcircled{2} \quad A \sim \mathcal{N}(100, 4^2), \quad B \sim \mathcal{N}(90, 3^2)$$

$$a) \quad \boxed{C := A - B} \quad C \sim \mathcal{N}\left(\underbrace{10}_\mu, \underbrace{4^2 + 3^2}_{\sigma^2 = 25}\right)$$

$$\begin{aligned} b) \quad P(A > B) &= P(C > 0) = P\left(\frac{C - 10}{5} > \frac{0 - 10}{5}\right) \\ &= 1 - \Phi(-2) = \Phi(2) = 0.9772 \end{aligned}$$

$$\begin{aligned} c) \quad P(B \text{ IS PROFITABLE TODAY}) &= P(B \geq 92) = \\ &= P\left(\frac{B - 90}{3} \geq \frac{92 - 90}{3}\right) = 1 - \Phi\left(\frac{2}{3}\right) = 1 - 0.7486 = \\ &= 0.2514 =: p \end{aligned}$$

$X :=$ NUMBER OF PROFITABLE DAYS
THIS YEAR

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$$X \sim \text{BIN}(365, p)$$

DE MOIVRE - LAPLACE

$$P(X \geq 100) = P\left(\frac{X - 365 \cdot p}{\sqrt{365 \cdot p \cdot (1-p)}} \geq \frac{100 - 365 \cdot p}{\sqrt{365 \cdot p \cdot (1-p)}}\right) \approx$$

$$1 - \Phi\left(\frac{100 - 365 \cdot p}{\sqrt{365 \cdot p \cdot (1-p)}}\right) = 1 - \Phi(0.99) = 0.1611$$

$$100 - 365 \cdot p = 8.239$$

$$365 \cdot p \cdot (1-p) = 68.69$$

$$\sqrt{365 \cdot p \cdot (1-p)} = 8.288$$

① a) $N(5, 5)$

b) $P(B < A) = P(B - A < 0) = P\left(\frac{B - A - 5}{5} < \frac{-5}{5}\right) =$

$\Phi(-1) = 1 - \Phi(1) = 1 - 0.8413 = 0.1587$

c) $P(\text{NOT CARE}) = P(A < 2) = \Phi\left(\frac{2}{5}\right) = \underbrace{0.7486}_P$

$X \sim \text{BIN}(30, p)$

$P(X \geq 25) = P\left(\frac{X - 30 \cdot p}{\sqrt{30 \cdot p \cdot (1-p)}} \geq \frac{25 - 30 \cdot p}{\sqrt{30 \cdot p \cdot (1-p)}}\right) =$

$= 1 - \Phi\left(\frac{25 - 30 \cdot p}{\sqrt{30 \cdot p \cdot (1-p)}}\right) = 1 - \Phi(1.07) = 1 - 0.8577 = 0.1423$

② a) $(1 - e^{-\frac{60}{65}}) \cdot (1 - e^{-\frac{60}{70}}) = 0.3469$

b) $f(x, y) = \frac{1}{70} \cdot e^{-\frac{1}{70} \cdot x} \cdot \frac{1}{65} \cdot e^{-\frac{1}{65} \cdot y}$

c) $\frac{\frac{1}{65}}{\frac{1}{65} + \frac{1}{70}} = 0.5185$