Tools of Modern Probability

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homework 1, fall 2021

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1.1 Calculate $f(a,b,c) := \int_{-\infty}^{\infty} e^{ax^2+bx+c}$ for all $a, b, c \in \mathbb{R}$, using the special case $a = -\frac{1}{2}$, b=c=0.

1.2 Calculate the (d-dimensional) volume of the ball $B_r^d := \{x \in \mathbb{R}^d \,|\, |x| \leq r\}$ (as a function of d and r). *(Hint: the volume is the integral of the indicator. The indicator is spherically symmetric.)*

1.3 Describe the asymptotic behaviour of $I_n := \int_0^2 (2x - x^2)^n \,\mathrm{d}x$ as $n \to \infty$.