2. PÓT ZH MEGOLDÁSOK

12.13.19

FÜGGETLEN ÉS AZONOS ELOSZLÁSÚ

1. $A_i \sim N(22, 3^2)$ IID

$B_i \sim N(23, 4^2)$ IID

$$A = A_1 + A_2 + A_3 + A_4 \sim N(\underbrace{4 \cdot 22}_{88}, \underbrace{4 \cdot 3^2}_{36})$$

$$B = B_1 + B_2 + B_3 + B_4 \sim N(\underbrace{4 \cdot 23}_{92}, \underbrace{4 \cdot 4^2}_{64})$$

a) $$\mathbb{P}(B > A) = \mathbb{P}(B - A > 0)$$

$$B - A \sim N(92 - 88, 36 + 64)$$

$$= \mathbb{P}\left(\frac{(B-A) - 4}{10} > \frac{-4}{10}\right)$$

$$= 1 - \Phi\left(\frac{-4}{10}\right) = \Phi\left(\frac{4}{10}\right) \approx 0.6554$$

b) Egy negyed:

$$\mathbb{P}(A_i > B_i) = \mathbb{P}(A_i - B_i > 0)$$

$$A_i - B_i \sim N(22 - 23, 9 + 16)$$

$$= \mathbb{P}\left(\frac{(A_i - B_i) + 1}{5} > \frac{1}{5}\right)$$

$$= 1 - \Phi\left(\frac{1}{5}\right) \approx 1 - 0.5793 = p$$

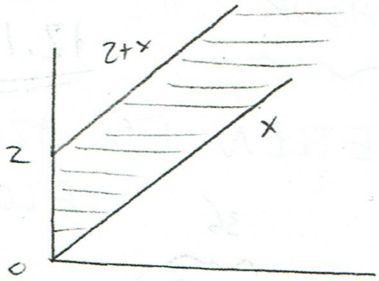
$\mathbb{P}(X \geq 2) = ?$ Ahol $X \sim Bin(4, p)$

$$= 1 - \mathbb{P}(X = 0) - \mathbb{P}(X = 1)$$

$$= 1 - (1-p)^4 - 4p(1-p)^3$$

$$\approx 0.5602$$

(2)



$$2 \geq y - x \geq 0$$

$$2 + x \geq y \geq x$$

a) $$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = \int_0^{\infty}\int_x^{2+x} Ae^{-2x}\,dy\,dx$$

$$= \int_0^{\infty} Ae^{-2x}\left(\int_x^{2+x} 1\,dy\right)dx$$

$$= \int_0^{\infty} 2Ae^{-2x}\,dx$$

$$= 2A\left[\frac{e^{-2x}}{-2}\right]_0^{\infty} = 2A\left(0 - \left(\frac{1}{-2}\right)\right)$$

$$= A = 1$$

b) $$f_X(x) = \begin{cases} \int_x^{2+x} e^{-2x}\,dy = 2e^{-2x} & \text{ha } x \geq 0 \\ 0 & \text{ha } x < 0 \end{cases}$$

$$f_Y(y) = \begin{cases} \int_{y-2}^{y} e^{-2x}\,dx = \left[\frac{e^{-2x}}{-2}\right]_{y-2}^{y} = \left(e^{-2(y-2)} - e^{-2y}\right)\frac{1}{2} & \text{ha } y \geq 2 \\ \int_0^{y} e^{-2x}\,dx = \left[\frac{e^{-2x}}{-2}\right]_0^{y} = \left(1 - e^{-2y}\right)\frac{1}{2} & \text{ha } 0 \leq y \leq 2 \\ 0 & \text{ha } y \leq 0 \end{cases}$$

c) $\mathbb{P}(Y < X + 1 \mid X = 2) = \mathbb{P}(Y < 3 \mid X = 2)$

$$= \int_{-\infty}^{3} f_{Y|X}(y, 2)\, dy = \int_{-\infty}^{3} \frac{f(2,y)}{f_X(2)}\, dy = \int_{2}^{3} \frac{e^{-4}}{2e^{-4}}\, dy$$

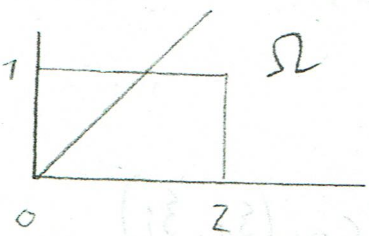
$\nwarrow y \geq x$

$$= \frac{1}{2}$$

<u>MEGJ</u>: $Y \mid X = x \sim UNI(x, x+2)$

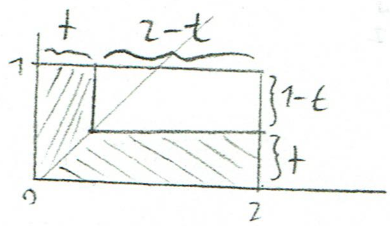
$Y \mid X = 2 \sim UNI(2, 4)$

(3)



$\mathbb{P}(Z \leq t) = ?$

$Ter(\Omega) = 2$



Ha $0 \leq t \leq 1$ akkor $\mathbb{P}(Z \leq t) = \dfrac{Ter(\text{▧})}{Ter(\Omega)}$

$$= \frac{2 - (1-t)(2-t)}{2}$$

ELOSZLÁS FGV.

$$\mathbb{P}(Z \leq t) = \begin{cases} 0 & \text{ha } t \leq 0 \\ 1 - \dfrac{(1-t)(2-t)}{2} & \text{ha } 0 \leq t \leq 1 \\ 1 & \text{ha } t \geq 1 \end{cases}$$

$(F_Z(t))$

SŰRŰSÉG FGV.

$$f_Z(t) = \frac{d}{dt} F_Z(t) = \begin{cases} 3/2 - t & \text{ha } 0 \leq t \leq 1 \\ 0 & \text{ha } t \geq 1 \text{ vagy } t \leq 0 \end{cases}$$

$$\mathbb{E}(Z) = \int_0^1 t(3/2 - t)\, dt = \frac{3}{2}\left[\frac{t^2}{2}\right]_0^1 - \left[\frac{t^3}{3}\right]_0^1 = \frac{3}{4} - \frac{1}{3} = \underline{\underline{\frac{5}{12}}}$$

(4) C_i = i. érmedobás

$\xi_1 \equiv 1$ ha $C_1 = F$

$i \geq 2 \quad \xi_i = \begin{cases} 1 & \text{ha } C_{i-1} = I, \ C_i = F \\ 0 & \text{különben} \end{cases}$

$$X = \xi_1 + \sum_{i=2}^{10} \xi_i$$

$$\mathbb{E}(X) = \mathbb{E}(\xi_1) + \sum_{i=2}^{10} \mathbb{E}(\xi_i) = \underset{\mathbb{P}(\xi_1 = 1)}{\frac{1}{2}} \quad 9\,\mathbb{P}(\xi_i = 1) \ \ (i \geq 2)$$

$$= \frac{1}{2} + 9 \cdot \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{11}{4}$$

BÓNUSZ

$$\mathrm{Var}(X) = \mathrm{Cov}(X, X) = \mathrm{Cov}(\xi_1, \xi_1) + 2\sum_{i=2}^{10} \mathrm{Cov}(\xi_1, \xi_i)$$

$$+ \underbrace{\sum_{i=2}^{9} \sum_{j=2}^{9} \mathrm{Cov}(\xi_i, \xi_j)}_{} = ?$$

Ha $i = j$, $\mathrm{Cov}(\xi_i, \xi_j) = p - p^2 \qquad \left(p = \frac{1}{2} \cdot \frac{1}{2}\right)$

Ha $|i-j| = 1 \quad \mathrm{Cov}(\xi_i, \xi_j)$

$$= \underbrace{\mathbb{E}(\xi_i \xi_j)}_{0} - \underbrace{\mathbb{E}(\xi_j)}_{p}\underbrace{\mathbb{E}(\xi_j)}_{p}$$

Ha $|i-j| \geq 2 \quad \mathrm{Cov}(\xi_i, \xi_j) = 0$ mert függetlenek

$$\mathrm{Cov}(\xi_1, \xi_1) = \frac{1}{2} - \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\mathrm{Cov}(\xi_1, \xi_2) = \underbrace{\mathbb{E}(\xi_1 \xi_2)}_{0} - \underbrace{\mathbb{E}(\xi_1)}_{1/2}\underbrace{\mathbb{E}(\xi_2)}_{p} = -\frac{1}{2}p$$

$\mathrm{Cov}(\xi_1, \xi_i) = 0$ Minden más i-re

$$\mathrm{Var}(X) = \frac{1}{4} - 2 \cdot \left(\frac{1}{2}p\right) + 9(p - p^2) + 2 \cdot 8(0 - p^2) = \frac{11}{16}$$