

NAME: NEPTUN-CODE:

Probability theory 1 exam, 2024. Feb. 9.

Working time: 100 min. Only simple, non-programmable calculators are allowed, standard normal distribution table on the other side.

The achievable maximum score (with the Bonus exercise) is 110 points, but we consider 100 points as 100%.

- T. 1.** (a) (1+2+4 points) State the inclusion-exclusion formula for two events, three events, and n events.
(b) (10 points) A group of n men went to have dinner in a restaurant. They left their hats in the wardrobe. After having dinner and drinking some wine, they took their hats out of the wardrobe completely at random. What the probability that at least one member of the group wore his own hat on the way home? Calculate the limit of this probability in the $n \rightarrow \infty$ limit.
- T. 2.** (a) (1+1+3+3 points) Define the geometric distribution based on its intuitive meaning. Calculate its probability mass function, its expected value, and its variance.
(b) (2+3+4 points) Define the hypergeometric distribution based on its intuitive meaning. Calculate its probability mass function and its expected value.
- Bonus:** (10 points) Calculate the variance of the hypergeometric distribution.
- T. 3.** (5+5+6 points) State and prove Markov's inequality, Chebyshev's inequality and the weak law of large numbers.
- P. 1.** The expected number of car accidents per day in the 11th district of Budapest is ten. The probability that the drivers involved in the accident will be able to agree on who is responsible is 0.6. If the drivers cannot agree, they call the police for an on-site inspection.
(a) (4 points) What distribution should we use to model the number of car accidents per day? Why?
(b) (6 points) What is the probability that the police will be called for at least three car accidents tomorrow?
(c) (7 points) Yesterday the police were called to the scenes of five car accidents. What is the probability that altogether ten car accidents occurred in the district yesterday?
- P. 2.** (4+3+5+5 points) The joint density function of X and Y is $f(x, y) = \frac{1}{y} e^{-(y+x/y)} \mathbb{1}[x > 0, y > 0]$.
(a) Determine the density function of Y . (b) Determine the expected value of Y . (c) Determine the conditional density function of X under the condition $Y = y$. (d) Determine the expected value of X .
- P. 3.** Mr. X travels to work by train and long-distance bus. On the way to work, he misses the connection with a probability of $1/5$, and on the way home with he misses the connection with probability of $1/4$. Let us assume that there are 220 working days per year, what is the approximate probability that Mr. X
(a) (6 points) will miss the connection at least forty times this year on his way to work?
(b) (10 points) will miss the connection more times on the way home than he will on the way to work next year?

