$\boxed{\text{ELM 1}}$ a) $\mathbb{P}(B|A) = \dfrac{\mathbb{P}(B \cap A)}{\mathbb{P}(A)}$

b) $\mathbb{P}(B_1 \cap B_2 | A) = \mathbb{P}(B_1|A) \cdot \mathbb{P}(B_2|A)$

c) $\mathbb{P}(\text{2023-BAN BAL.} \mid \text{2022-BEN BAL.}) =$

$$= \frac{\mathbb{P}(\text{2022-BEN ÉS 2023-BAN BAL.})}{\mathbb{P}(\text{2022-BEN BAL.})} = \frac{\mathbb{P}(B_{22} \cap B_{23})}{\mathbb{P}(B_{22})} =$$

$$= \frac{\mathbb{P}(B_{22} \cap B_{23} | \text{ÓVATOS}) \cdot 0.8 + \mathbb{P}(B_{22} \cap B_{23} | \text{ÓVATLAN}) \cdot 0.2}{\mathbb{P}(B_{22} | \text{ÓVATOS}) \cdot 0.8 + \mathbb{P}(B_{22} | \text{ÓVATLAN}) \cdot 0.2} =$$

$$= \frac{(0.05)^2 \cdot 0.8 + (0.15)^2 \cdot 0.2}{(0.05) \cdot 0.8 + (0.15) \cdot 0.2} = 0.092857 = \frac{13}{140}$$

B_{22} ÉS B_{23} FELT. FÜGGETLEN, HA $\begin{cases} \text{ÓVATOS} \\ \text{ÓVATLAN} \end{cases}$

$\boxed{\text{ELM 2}}$ a) $\varphi(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2}$

POLÁR

$$\left(\int_{-\infty}^{\infty} \varphi(x)\,dx\right)^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \varphi(x)\varphi(y)\,dx\,dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{2\pi} e^{-(x^2+y^2)/2}\,dx\,dy =$$

$$= \int_0^{2\pi}\int_0^{\infty} \frac{1}{2\pi} e^{-r^2/2} \cdot r\,dr\,d\varphi = \int_0^{\infty} e^{-r^2/2} \cdot r\,dr =$$

$$\left[-e^{-r^2/2}\right]_0^{\infty} = 1 \quad \text{TEHÁT} \quad \int_{-\infty}^{\infty} \varphi(x)\,dx = 1$$

1. OLDAL

$\boxed{\text{ELM 2}}$ b) $X \sim N(0,1)$ $\boxed{\varphi(x) = \varphi(-x)}$

$$E(X) = \int_{-\infty}^{\infty} x \cdot \varphi(x)\, dx \overset{!}{=} 0$$ HISZ PÁRATLAN FÜGGVÉNYT INTEGRÁLUNK

$$Var(X) = E(X^2) - 0^2 = \int_{-\infty}^{\infty} x^2 \cdot \varphi(x)\, dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \underbrace{x}_{g} \cdot \underbrace{x \cdot e^{-x^2/2}}_{f'}\, dx =$$

PARCIÁLIS

$$= \frac{1}{\sqrt{2\pi}} \Big[f \cdot g \Big]_{-\infty}^{\infty} - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f \cdot g'\, dx = \frac{1}{\sqrt{2\pi}} \Big[\left(-e^{-x^2/2}\right) \cdot x \Big]_{-\infty}^{\infty} +$$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2}\, dx = 1$$

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c) $X_i :=$ i-EDIK MÉRÉS EREDMÉNYE, $X_i \sim N(\mu, 4)$

HA n MÉRÉST VÉGEZ, AKKOR A MINTAÁTLAG:

$$Y_n = \frac{X_1 + \ldots + X_n}{n} = \frac{S_n}{n}. \quad S_n \sim N(n \cdot \mu, n \cdot 4)$$

$$Y_n \sim N\left(\mu, \frac{4}{n}\right) \qquad 0.95 \overset{\text{KELL}}{\leq} P(|Y_n - \mu| \leq 0.5) =$$

$$= P(-0.5 \leq Z_n \leq 0.5) = \Phi\left(\frac{0.5}{\sqrt{4/n}}\right) - \Phi\left(\frac{-0.5}{\sqrt{4/n}}\right) =$$

$Z_n := Y_n - \mu$, $Z_n \sim N\left(0, \frac{4}{n}\right)$

$$= 2 \cdot \Phi\left(\frac{\sqrt{n}}{4}\right) - 1 \overset{\text{KELL}}{\geq} 0.95 \iff \Phi\left(\frac{\sqrt{n}}{4}\right) \geq 0.975$$

$$\iff \frac{\sqrt{n}}{4} \geq 1.96 \iff n \geq 61.46 \iff \boxed{n = 62}$$

2. OLDAL

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GYAK 1 HULLÓCSILLAGOK 2 INTENZITÁSÚ POISSON PONTFOLYAMATOT ALKOTNAK

$$\mathbb{E}(X) \overset{\text{TORONY}}{=} \mathbb{E}(\mathbb{E}(X|U)) = \mathbb{E}(2\cdot U) = 2$$

$$\mathrm{Var}(X) \overset{\text{FELT. SZÓRÁS } \square}{=} \mathrm{Var}(\mathbb{E}(X|U)) + \mathbb{E}(\mathrm{Var}(X|U)) =$$

$$= \mathrm{Var}(2\cdot U) + \mathbb{E}(2U) = 4\cdot\frac{2^2}{12} + 2 = \frac{10}{3} = 3.\dot{3}$$

GYAK 2 $\mathrm{Corr}(X,Y) = \dfrac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)}\cdot\sqrt{\mathrm{Var}(Y)}} \overset{\text{SZIMM.}}{=} \dfrac{\mathrm{Cov}(X,Y)}{\mathrm{Var}(X)}$

$A_i := \{$AZ i-EDIK EMBER NAGYOBBAT DOBOTT AZ i+1-EDIKNÉL$\}$

$B_i := \{$ — " — — " — KISEBBET — " — — " — $\}$

$$X_i := \mathbb{1}[A_i],\quad Y_i := \mathbb{1}[B_i],\quad X = \sum_{i=1}^{m} X_i,\quad Y = \sum_{i=1}^{m} Y_i$$

$$\mathrm{Cov}(X,Y) = \sum_{i,j=1}^{m} \mathrm{Cov}(X_i,Y_j) =$$

$\boxed{i=j}$ $\mathrm{Cov}(X_i,Y_i) = \underbrace{\mathbb{P}(A_i\cap B_i)}_{0} - \underbrace{\mathbb{P}(A_i)}_{\frac{1}{3}}\cdot\underbrace{\mathbb{P}(B_i)}_{\frac{1}{3}} = -\frac{1}{9}$

$\boxed{j=i+1}$ $\mathrm{Cov}(X_i,Y_{i+1}) = \underbrace{\mathbb{P}(A_i\cap B_{i+1})}_{5/27} - \underbrace{\mathbb{P}(A_i)}_{1/3}\cdot\underbrace{\mathbb{P}(B_{i+1})}_{1/3} = \frac{2}{27}$

$\boxed{j=i-1}$ $\mathrm{Cov}(X_i,Y_{i-1}) \overset{\text{SZIMM.}}{=} \frac{2}{27}$

$\boxed{|i-j|\geq 2}$ $\Longrightarrow \mathrm{Cov}(X_i,Y_j) = 0$ FÜGGETLENSÉG

3. OLDAL

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$$☺ = n\cdot\left(-\frac{1}{9}\right) + 2\cdot n\cdot\frac{2}{27} = \frac{n}{27}$$

$$\mathrm{Var}(X) = \sum_{i,j=1}^{n} \mathrm{Cov}(X_i, X_j) = ★$$

$$\boxed{i=j} \quad \mathrm{Cov}(X_i, X_j) = \frac{1}{3}\cdot\left(1-\frac{1}{3}\right) = \frac{2}{9}$$

$$\boxed{i=j\pm 1} \quad \mathrm{Cov}(X_i, X_j) = \underbrace{\mathbb{P}(A_i \cap A_{i+1})}_{1/27} - \left(\frac{1}{3}\right)^2 = -\frac{2}{27}$$

$$\boxed{|i-j| \geq 2} \Rightarrow \mathrm{Cov}(X_i, X_j) = 0$$

FÜGGETLENSÉG

$$★ = n\cdot\frac{2}{9} + 2\cdot n\cdot\left(-\frac{2}{27}\right) = n\cdot\frac{2}{27}$$, ÍGY

$$\mathrm{Corr}(X, Y) = \frac{n/27}{2n/27} = \frac{1}{2}$$

☹

$\boxed{\text{GYAK 3}}$ $\quad U \sim \mathrm{UNI}[0, \ell] \quad X = U^2 + (\ell - U)^2 =$

$$= U^2 + \ell^2 - 2\ell U + U^2 = 2U^2 - 2\ell U + \ell^2 =$$

$$= 2\cdot\left(U - \frac{\ell}{2}\right)^2 + \frac{\ell^2}{2} \qquad \mathbb{P}\left(\frac{\ell^2}{2} \leq X \leq \ell^2\right) = 1$$

$$x \in \left[\ell^2/2, \ell^2\right] \Rightarrow \mathbb{P}(X \leq x) = \mathbb{P}\left(2\cdot\left(U-\frac{\ell}{2}\right)^2 + \frac{\ell^2}{2} \leq x\right) =$$

$$= \mathbb{P}\left(\left(U - \frac{\ell}{2}\right)^2 \leq \frac{x}{2} - \frac{\ell^2}{4}\right) = \mathbb{P}\left(\left|U - \frac{\ell}{2}\right| \leq \sqrt{\frac{x}{2} - \frac{\ell^2}{4}}\right) =$$

$$= \frac{2\cdot\sqrt{\frac{x}{2} - \frac{\ell^2}{4}}}{\ell} = F(x), \text{ HA } \frac{\ell^2}{2} \leq x \leq \ell^2$$

FOLYT. KÖV.

⚹ SŰ.FV.-E: $f(x) = F'(x) = \frac{d}{dx}\left(\frac{2\cdot\sqrt{\frac{x}{2}-\frac{\ell^2}{4}}}{\ell}\right) =$

$= \frac{d}{dx}\left(\frac{\sqrt{2x-\ell^2}}{\ell}\right) = \frac{1}{2\cdot\sqrt{2x-\ell^2}}\cdot 2\cdot\frac{1}{\ell} =$

$= \frac{1}{\ell}\cdot\frac{1}{\sqrt{2x-\ell^2}}$, HA $\frac{\ell^2}{2} \le x \le \ell^2$, AMÚGY $f(x)=0$.

<u>BÓNUSZ</u>: $P_{n,k} := \mathbb{P}$(ÁDÁM KÖRE k FŐBŐL ÁLL, AMIKOR n FŐS A KÖZÖSSÉG)

<u>TÉTEL</u>: $P_{n,k} = \frac{1}{n-1}$, HA $k = 1, 2, \ldots, n-1$

<u>BIZ</u>: n-RE VONATKOZÓ TELJES INDUKCIÓVAL

$\boxed{n=2}$ ESETÉN $P_{2,1} = 1$ ✓

ÁLTALÁNOS $\boxed{n \to n+1}$:

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$P_{n+1,1} = P_{n,1}\cdot\frac{n-1}{n} = \frac{1}{n-1}\cdot\frac{n-1}{n} = \frac{1}{n} = \frac{1}{(n+1)-1}$ ✓

$2 \le k \le n$: $P_{n+1,k} = P_{n,k-1}\cdot\frac{k-1}{n} + P_{n,k}\cdot\frac{n-k}{n} =$

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$= \frac{1}{n-1}\cdot\frac{k-1}{n} + \frac{1}{n-1}\cdot\frac{n-k}{n} = \frac{1}{n}$ ✓