$\boxed{\text{ELM 1.}}$ a) $X \sim GEO(p)$, HA $\mathbb{P}(X = k) = (1-p)^{k-1} \cdot p$, $k = 1, 2, 3, \ldots$

ÖRÖKIF ∃Ú: $\mathbb{P}(X > k + l \mid X > l) = \mathbb{P}(X > k)$

BIZ: $\mathbb{P}(X > k) = \sum_{l=k+1}^{\infty} (1-p)^{l-1} \cdot p = p \cdot (1-p)^k \cdot \sum_{l=0}^{\infty} (1-p)^l = (1-p)^k$

$$\mathbb{P}(X > k + l \mid X > l) = \frac{(1-p)^{k+l}}{(1-p)^l} = (1-p)^k = \mathbb{P}(X > k)$$

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b) $\mathbb{E}(X) = \sum_{k=1}^{\infty} \mathbb{P}(X \geq k) = \sum_{k=0}^{\infty} \mathbb{P}(X > k) =$

$$= \sum_{k=0}^{\infty} (1-p)^k = \frac{1}{1-(1-p)} = \frac{1}{p}$$

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c) $\mathbb{P}(X = 5 \mid X + Y = 13) = \dfrac{(1-p)^4 \cdot p \cdot (1-p)^7 \cdot p}{\sum_{k=1}^{12} (1-p)^{k-1} \cdot p \cdot (1-p)^{12-k} \cdot p} =$

$$= \frac{(1-p)^{11} \cdot p^2}{\sum_{k=1}^{12} (1-p)^{11} \cdot p^2} = \frac{1}{12}$$

$\boxed{\text{ELM 2.}}$ a) $f_{Z_1}(z) = \int_{-\infty}^{\infty} f(x, z)\, dx \qquad f_{X|Z_1}(x|z) = \dfrac{f(x,z)}{f_{Z_1}(z)}$

$$\mathbb{E}(X \mid Z_1 = z) = \int_{-\infty}^{\infty} x \cdot f_{X|Z_1}(x|z)\, dx$$

b) $f_X(x) = \dfrac{1}{\sqrt{2\pi} \cdot \sigma} \cdot \exp\left(-\dfrac{(x-\mu)^2}{2\sigma^2}\right)$

$$f_{Z_1|X}(z|x) = \frac{1}{\sqrt{2\pi}} \cdot \exp\left(-\frac{(z-x)^2}{2}\right)$$

$$f(x,z) = f_X(x) \cdot f_{Z|X}(z|x) = \frac{1}{2\pi} \cdot \frac{1}{\sigma} \cdot \exp\left(-\frac{(x-\mu)^2}{2\sigma^2} - \frac{(z-x)^2}{2}\right)$$

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$$Z_1 \sim \mathcal{N}(\mu, \sigma^2+1), \quad f_{Z_1}(z) = \frac{1}{\sqrt{2\pi}\,\sqrt{\sigma^2+1}} \cdot \exp\left(-\frac{(z-\mu)^2}{2\cdot(\sigma^2+1)}\right)$$

$$f_{X|Z_1}(x|z_1) = \frac{f(x,z)}{f_{Z_1}(z)} = \frac{\frac{1}{2\pi}\cdot\frac{1}{\sigma}\cdot\exp\left(-\frac{(x-\mu)^2}{2\sigma^2} - \frac{(z-x)^2}{2}\right)}{\frac{1}{\sqrt{2\pi}}\cdot\frac{1}{\sqrt{\sigma^2+1}}\cdot\exp\left(-\frac{(z-\mu)^2}{2\cdot(\sigma^2+1)}\right)} =$$

$$C\cdot\exp\left(-\frac{1}{2}\cdot\left(\frac{(x-\mu)^2}{\sigma^2} + (z-x)^2 - \frac{(z-\mu)^2}{\sigma^2+1}\right)\right) =$$

$$\hat{C}\cdot\exp\left(-\frac{1}{2}\cdot\left(\frac{x^2}{\sigma^2} - \frac{2x\mu}{\sigma^2} - 2z\cdot x + x^2\right)\right) = \star$$

$$= \frac{1+\sigma^2}{\sigma^2}\cdot x^2 - 2\cdot\left(\frac{\mu+\sigma^2\cdot z}{\sigma^2}\right)\cdot x = \frac{1+\sigma^2}{\sigma^2}\cdot\left(x^2 - 2\cdot\frac{\mu+\sigma^2\cdot z}{1+\sigma^2}\cdot x\right)$$

$$\star = \tilde{C}\cdot\exp\left(-\frac{\left(x - \frac{\mu+\sigma^2\cdot z}{1+\sigma^2}\right)^2}{2\cdot\left(\frac{\sigma^2}{1+\sigma^2}\right)}\right), \text{ ÍGY}$$

X FELTÉTELES ELOSZLÁSA A $Z_1 = z$ FELTÉTEL MELLETT: $\mathcal{N}\left(\mu\cdot\frac{1}{1+\sigma^2} + z\cdot\frac{\sigma^2}{1+\sigma^2},\ \frac{\sigma^2}{1+\sigma^2}\right)$

ÍGY $\mathbb{E}(X \mid Z_1 = z) = \frac{1}{1+\sigma^2}\cdot\mu + \frac{\sigma^2}{1+\sigma^2}\cdot z$

$\boxed{\text{ELM 3.}}$ a) $f(x) = F'(x)$, $F(x) = \int_{-\infty}^{x} f(y)\,dy$

b) $G(y) = \mathbb{P}(\mathfrak{Y} < y) = \mathbb{P}(\Psi(\mathfrak{X}) < y) =$

$\mathbb{P}(\mathfrak{X} < \Psi^{-1}(y)) = F(\Psi^{-1}(y))$

$g(y) = G'(y) = F'(\Psi^{-1}(y)) \cdot (\Psi^{-1})'(y) =$

$= f(\Psi^{-1}(y)) \cdot \dfrac{1}{\Psi'(\Psi^{-1}(y))}$

$\mathfrak{X}$ ABSZ. FOLYT

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c) $G(y) = \mathbb{P}(\mathfrak{Y} < y) = \mathbb{P}(1 - 2\mathfrak{X} < y) = \mathbb{P}\left(\frac{1-y}{2} < \mathfrak{X}\right) =$

$= \mathbb{P}\left(\frac{1-y}{2} \le \mathfrak{X}\right) = 1 - \mathbb{P}\left(\mathfrak{X} < \frac{1-y}{2}\right) = 1 - F\left(\frac{1-y}{2}\right)$

$g(y) = G'(y) = -F'\left(\frac{1-y}{2}\right) \cdot \left(-\frac{1}{2}\right) = \frac{1}{2} \cdot f\left(\frac{1-y}{2}\right)$

$\boxed{\text{GYAK 1}}$ $\mathbb{P}$(FEHÉR VOLT | FEHÉRNEK LÁTTA) =

$$\frac{\mathbb{P}(\text{FEHÉR VOLT ÉS FEHÉRNEK LÁTTA})}{\mathbb{P}(\text{FEHÉRNEK LÁTTA})} = \frac{(0.15)\cdot(0.8)}{(0.85)\cdot(0.2) + (0.15)\cdot(0.8)} =$$

$= 0.4138$

BÓNUSZ: $\mathbb{P}$(FEHÉR VOLT | FEHÉRNEK LÁTTÁK) =

$$= \frac{\mathbb{P}(\text{FEHÉR VOLT, MIND2EN FEHÉRNEK LÁTTÁK})}{\mathbb{P}(\text{MIND2-EN F-NEK LÁTTÁK})} = \frac{(0.15)\cdot(0.8)^2}{(0.85)\cdot(0.2)^2 + (0.15)\cdot(0.8)^2}$$

$= 0.7385$

$\boxed{\text{GYAK 2}}$ $\quad X \sim BIN(1000, 0.3)$, $\quad Y \sim BIN(1500, 0.3)$

$$\mathbb{P}\left(\left| \frac{X}{10^3} - 0.3 \right| < \left| \frac{Y}{1500} - 0.3 \right| \right) = \text{☺}$$

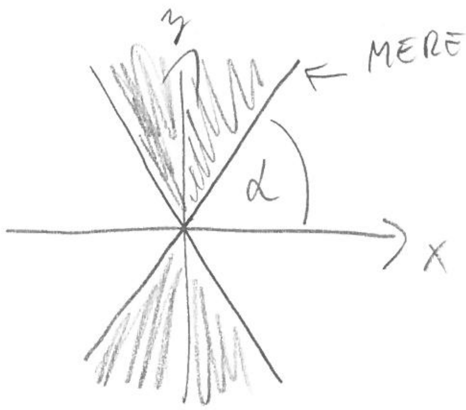
$$X^* := \frac{X - 10^3 \cdot 0.3}{\sqrt{10^3} \cdot \sqrt{0.3 \cdot (1-0.3)}} \qquad Y^* := \frac{Y - 1500 \cdot 0.3}{\sqrt{1500} \cdot \sqrt{0.3 \cdot (1-0.3)}}$$

$$\text{☺} = \mathbb{P}\left(\left| \frac{X - 10^3 \cdot 0.3}{10^3} \right| < \left| \frac{Y - 1500 \cdot 0.3}{1500} \right| \right) =$$

DE MOIVRE-LAPLACE

$$\mathbb{P}\left(\left| \frac{X^* \cdot \sqrt{10^3} \cdot \sqrt{0.3 \cdot 0.7}}{10^3} \right| < \left| \frac{Y^* \cdot \sqrt{1500} \cdot \sqrt{0.3 \cdot 0.7}}{1500} \right| \right) =$$

$$\mathbb{P}\left(\left| \frac{X^*}{\sqrt{10^3}} \right| < \left| \frac{Y^*}{\sqrt{1500}} \right| \right) = \mathbb{P}\left(|Y^*| > \sqrt{\frac{3}{2}} \cdot |X^*| \right) \approx$$



$\leftarrow$ MEREDEKSÉG $= \sqrt{\frac{3}{2}}$

FELTESSZÜK, HOGY X^*, Y^* F.A.E. $N(0,1)$

$$\alpha = \arctan\left(\sqrt{\frac{3}{2}}\right) = 0.886$$

$$\approx \frac{4 \cdot (\pi/2 - \alpha)}{2\pi} = \frac{\pi/2 - \alpha}{\pi/2} = 0.436$$

KÉTDIMENZIÓS STANDARD NORMÁLIS SZÖGE EGYENLETES ELOSZLÁSÚ $[0, 2\pi]$-n

$\boxed{\text{GYAK 3}}$ $A_i := \{$ i-EDIK PÖRGETÉSNÉL XAVÉR NYER $\}$

$B_i := \{$ —"— —"— YVETT NYER $\}$

$X_i := \mathbb{1}[A_i]$, Y_i: $\mathbb{1}[B_i]$ || $p := \mathbb{P}(A_i) = \frac{18}{37}$

$X = \sum_{i=1}^{20} X_i$, $Y = \sum_{i=1}^{20} Y_i$ || $q := \mathbb{P}(B_i) = \frac{12}{37}$

$X \sim BIN(20, p)$ $\mathbb{E}(X) = 20 \cdot p$, $Var(X) = 20 \cdot p \cdot (1-p)$

$Y \sim BIN(20, q)$ $\mathbb{E}(Y) = 20 \cdot q$, $Var(Y) = 20 \cdot q \cdot (1-q)$

$$Cov(X, Y) = \sum_{i,j=1}^{20} Cov(X_i, Y_j) \underset{\uparrow}{=} 20 \cdot (\mathbb{P}(A_i \cap B_i) - p \cdot q)$$

$\boxed{i \neq j}$: $Cov(X_i, Y_j) = 0$ (A PÖRGETÉSEK FÜGGETLENEK)

$\boxed{i = j}$: $Cov(X_i, Y_i) = \mathbb{P}(A_i \cap B_i) - p \cdot q$

$$Corr(X, Y) = \frac{Cov(X, Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}} = \frac{\mathbb{P}(A_i \cap B_i) - p \cdot q}{\sqrt{p \cdot (1-p)}\sqrt{q \cdot (1-q)}} = ☺$$

$$\mathbb{P}(A_i \cap B_i) = \mathbb{P}(19, 22, 25, 28, 31, 34) = \frac{6}{37}$$

$$☺ = \frac{(6/37) - \frac{18}{37} \cdot \frac{12}{37}}{\sqrt{\frac{18}{37} \cdot \frac{19}{37} \cdot \frac{12}{37} \cdot \frac{25}{37}}} = 0.01873$$