① $Y \sim N(\underbrace{10+10+10+17}_{47}, \underbrace{1^2+2^2+2^2+4^2}_{25})$

a) $P(Y<50) = P\left(\frac{Y-47}{5} < \frac{50-47}{5}\right) = \Phi\left(\frac{3}{5}\right) = 0.7257$

b) $P(Y>47) = P\left(\frac{Y-47}{5} > \frac{47-47}{5}\right) = 1-\Phi(0) = \frac{1}{2}$

$X \sim BIN(\underbrace{400}_{n}, \underbrace{\tfrac{1}{2}}_{p})$ $\quad n \cdot p = 200$, $\sqrt{n \cdot p \cdot (1-p)} = 10$

$P(X \le k_0) = P\left(\frac{X-200}{10} \le \frac{k_0-200}{10}\right) \overset{\text{DE MOIVRE}}{\approx} \Phi\left(\frac{k_0-200}{10}\right)$

$\Phi^{-1}(0.95) = 1.65$, ÍGY $\frac{k_0-200}{10} = 1.65$

TEHÁT $\boxed{k_0 = 217}$

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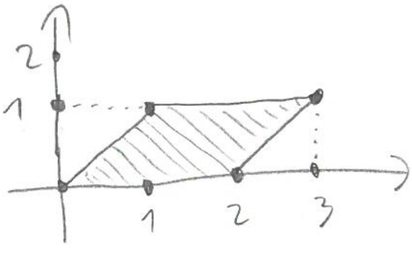
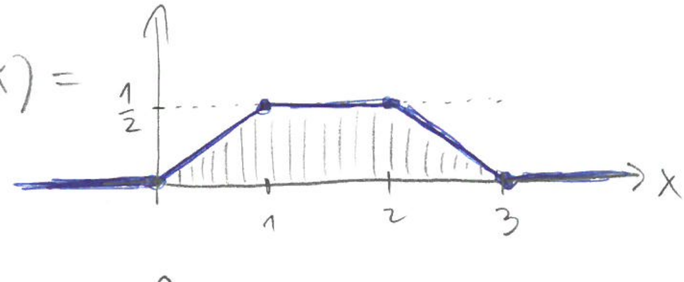
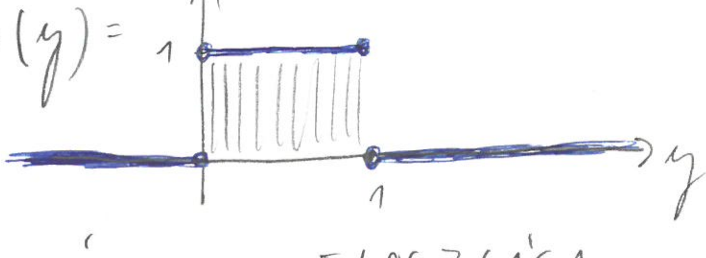
②



a) $f_X(x) =$



$f_Y(y) =$



b) HA $Y = \frac{1}{2}$, AKKOR X FELTÉTELES ELOSZLÁSA UNI$[\frac{1}{2}, \frac{5}{2}]$, ÍGY $P(X>1 \mid Y=\frac{1}{2}) = \frac{3}{4}$

c) $$\mathbb{E}(X+Y) = \underbrace{\mathbb{E}(X)}_{\frac{3}{2}} + \underbrace{\mathbb{E}(Y)}_{\frac{1}{2}} = 2$$

SZIMMETRIA → $\frac{3}{2}$ $\qquad$ $\frac{1}{2}$ ← SZIMMETRIA

MINDKÉT ESETBEN SZIMMETRIA MIATT KÖZÉPEN VAN A SÚLYPONT (AZAZ A VÁRHATÓ ÉRTÉK)

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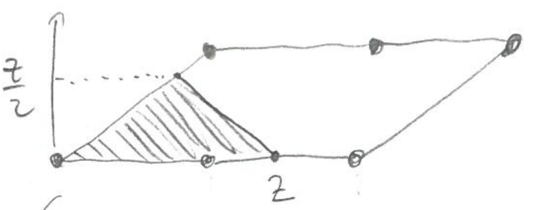
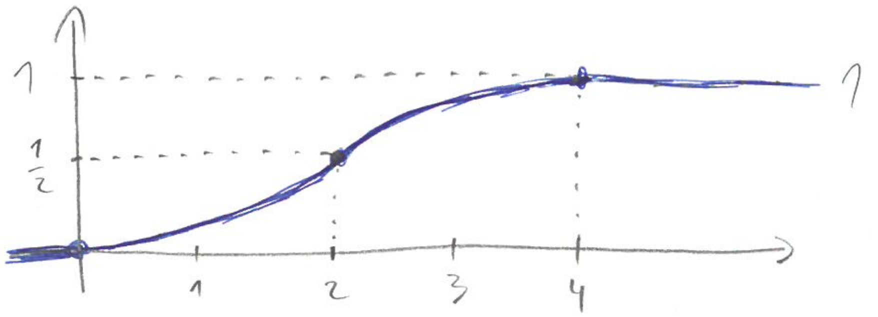
d) $$F(z) = \mathbb{P}(Z < z) = \begin{cases} 0, & \text{HA } z \le 0 \\ \text{☺}, & \text{HA } 0 \le z \le 2 \\ \text{☹}, & \text{HA } 2 \le z \le 4 \\ 1, & \text{HA } z \ge 4 \end{cases}$$



$$\text{☺} = \frac{\text{KEDVEZŐ TERÜLET}}{\text{ÖSSZES TERÜLET}} = \frac{\left(z \cdot \frac{z}{2}\right)/2}{2} = \frac{z^2}{8}$$

$$\text{☹} = 1 - \frac{\left((4-z)\cdot(4-z)/2\right)/2}{2} = 1 - \frac{(4-z)^2}{8}$$



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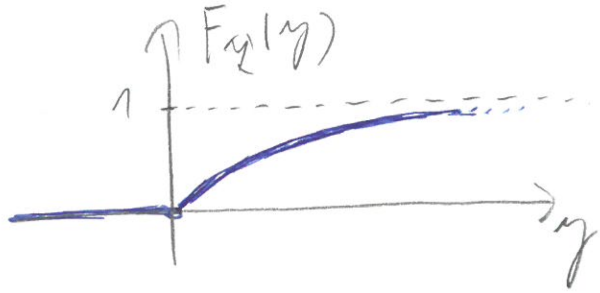
③ (a) $$F_Y(y) = \mathbb{P}(Y < y) = \mathbb{P}\left(e^{-2X} < y\right) \overset{\text{HA } y>0}{=}$$

$$\mathbb{P}(-2X < \ln(y)) = \mathbb{P}\left(X > -\tfrac{1}{2}\ln(y)\right) = 1 - \mathbb{P}\left(X < -\tfrac{1}{2}\ln(y)\right)$$

$$= 1 - F_X\left(-\tfrac{1}{2}\ln(y)\right) = 1 - \exp\left(-3 \cdot e^{\ln(y)}\right) = 1 - e^{-3y}$$

③ (a) $\mathbb{P}(Y>0)=1$, ÍGY $F_Y(y)=0$, HA $y<0$



$Y \sim EXP(3)$

ÖRÖKIFJÚ

(b) $\mathbb{P}(Y>1 \mid Y>\frac{1}{3}) \overset{\text{ÖRÖKIFJÚ}}{=} \mathbb{P}(Y>\frac{2}{3}) = e^{-3\cdot\frac{2}{3}} = e^{-2}$

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④ $X_i := \mathbb{1}[A_i]$, $A_i := \{$ i-EDIK LAP MEGFORDUL A KEZEMBEN $\}$

$X = \sum_{i=1}^{52} X_i$ a) $\mathbb{E}(X) = \sum_{i=1}^{52} \mathbb{P}(A_i) = 52\cdot\mathbb{P}(A_1)$,

$\mathbb{P}(A_1) = 1-\mathbb{P}($SOSEM LÁTOM A PIKK DÁMÁT$) =$

$= 1-\left(\frac{\binom{51}{5}}{\binom{52}{5}}\right)^{15} = 1-\left(\frac{47}{52}\right)^{15} = 0.78 =: p$, $\mathbb{E}(X) = 40.586$

b) $Var(X) = \sum_{i,j=1}^{52} Cov(X_i, X_j) =$

$= 52\cdot\underbrace{Var(X_1)}_{=p\cdot(1-p)} + 52\cdot 51\cdot\underbrace{Cov(X_1,X_2)}_{=\mathbb{P}(A_1\cap A_2)-p^2}$

$\mathbb{P}(A_1\cap A_2) \overset{\text{DE MORGAN}}{=} 1-\mathbb{P}(A_1^c\cup A_2^c) \overset{\text{SZITA}}{=} 1-\underbrace{\mathbb{P}(A_1^c)}_{1-p}-\underbrace{\mathbb{P}(A_2^c)}_{1-p}+\mathbb{P}(A_1^c\cap A_2^c)$

$\mathbb{P}(A_1^c\cap A_2^c) = \left(\frac{\binom{50}{5}}{\binom{52}{5}}\right)^{15} = \left(\frac{47\cdot 46}{52\cdot 51}\right)^{15}$