2019 NOV 28.
2. ZH (B)

(1) $Y \sim N(\underbrace{4\times 15+50}_{110},\ \underbrace{4\times 3^2+8^2}_{100})$

a) $P(Y<120) = P\left(\frac{Y-110}{10} < \frac{120-110}{10}\right) = \Phi(1) = \underline{0.8413}$

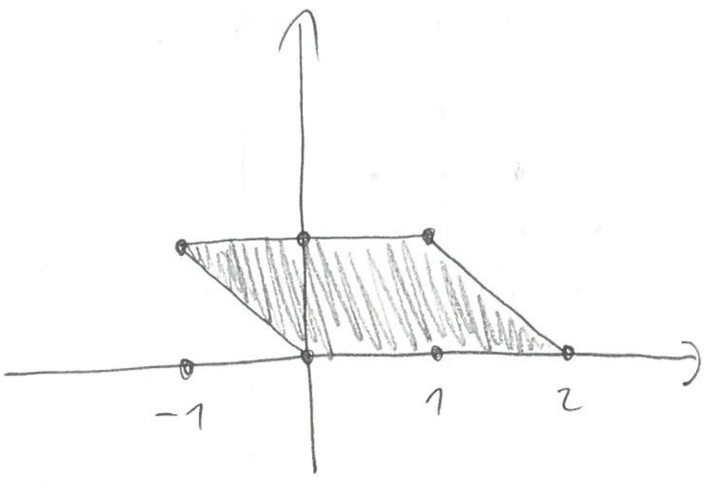
b) $P(Y>110) = P\left(\frac{Y-110}{10} > \frac{110-110}{10}\right) = 1-\Phi(0) = \frac{1}{2}$

$X \sim BIN(n,p)$: $n=900$, $p=\frac{1}{2}$, $n\cdot p = 450$, $\sqrt{n\cdot p\cdot(1-p)} = 15$

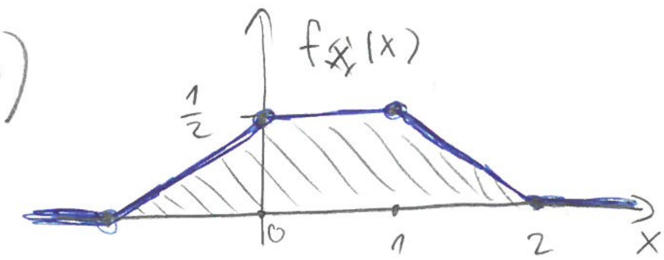
$P(X \le k_0) = P\left(\frac{X-450}{15} \le \frac{k_0-450}{15}\right) \overset{\text{DE MOIVRE}}{\approx} \Phi\left(\frac{k_0-450}{15}\right)$

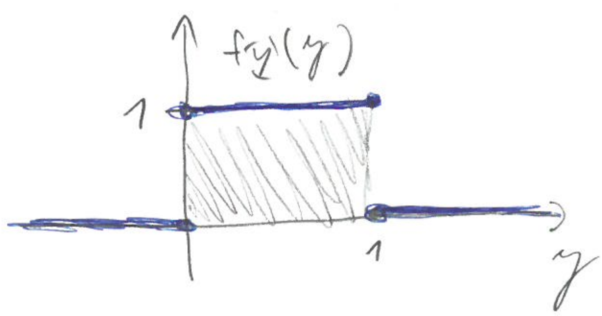
$\Phi^{-1}(0.96) = 1.76$, ÍGY $\frac{k_0-450}{15} = 1.76$: $\boxed{k_0 = 477}$

(2)



a)





b) HA $Y=\frac{1}{2}$, AKKOR X FELTÉTELES ELOSZLÁSA

UNI$\left[-\frac{1}{2}, \frac{3}{2}\right]$, ÍGY $P\left(X>1 \mid Y=\frac{1}{2}\right) = \frac{1}{4}$

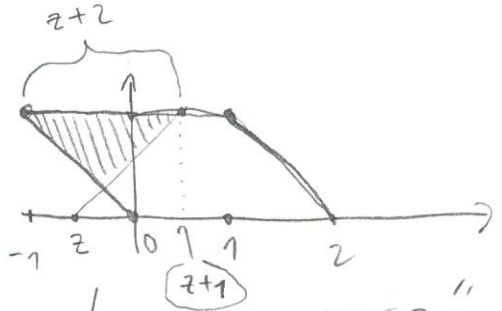
1.0CDAC

c) $\mathbb{E}(X - Y) = \underbrace{\mathbb{E}(X)}_{\frac{1}{2}} - \underbrace{\mathbb{E}(Y)}_{\frac{1}{2}} = 0$

SZIMMETRIA $\longrightarrow \frac{1}{2}$ $\qquad \frac{1}{2} \longleftarrow$ SZIMMETRIA

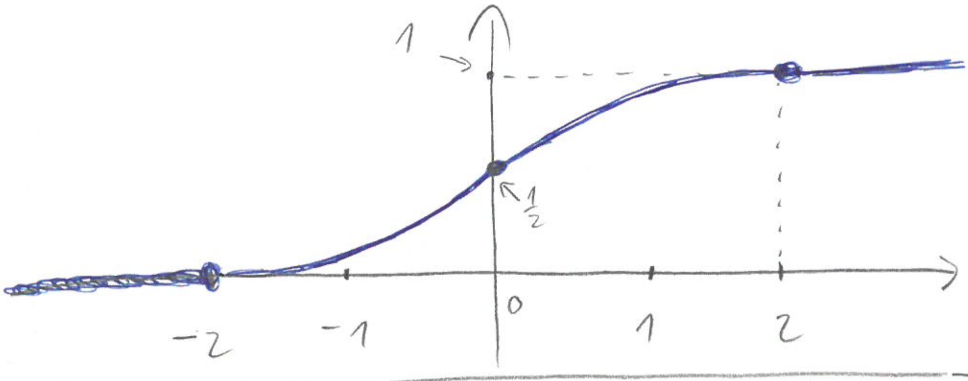
MINDKÉT ESETBEN SZIMMETRIA MIATT
KÖZÉPEN VAN A SÚLYPONT (AZAZ A VÁRHATÓ ÉRTÉK)

d) $F(z) = \mathbb{P}(Z_1 < z) = \begin{cases} 0, \text{ HA } & z \leq -2 \\ ☺, \text{ HA } & -2 \leq z \leq 0 \\ ☹, \text{ HA } & 0 \leq z \leq 2 \\ 1, \text{ HA } & z \geq 2 \end{cases}$



$☺ = \frac{\text{KEDVEZŐ TERÜLET}}{\text{ÖSSZES TERÜLET}} = \frac{\frac{1}{2} \cdot (z+2) \cdot \frac{z+2}{2}}{2} = \frac{(z+2)^2}{8}$

$☹ = 1 - \frac{\frac{1}{2} \cdot (2-z) \cdot \frac{2-z}{2}}{2} = 1 - \frac{(2-z)^2}{8}$

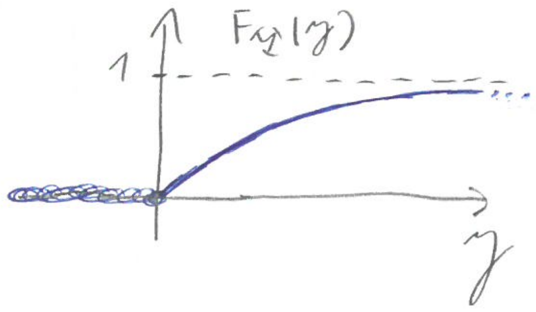


3) a) $F_Y(y) = \mathbb{P}(Y < y) = \mathbb{P}(e^{-3X} < y) \overset{\text{HA } y>0}{=\!=} \mathbb{P}(-3X < \ln(y)) =$

$= \mathbb{P}(X > -\frac{1}{3}\ln(y)) = 1 - \mathbb{P}(X < -\frac{1}{3}\ln(y)) = 1 - F_X(-\frac{1}{3}\ln(y)) =$

$= 1 - \exp(-2 \cdot e^{\ln(y)}) = 1 - e^{-2y}$

③ a) $\mathbb{P}(Y > 0) = 1$, ÍGY $F_Y(y) = 0$, HA $y < 0$



TEHÁT $Y \sim EXP(2)$

b) $\mathbb{P}\left(Y > 1 \mid Y > \frac{1}{2}\right) \overset{\text{ÖRÖKIFJÚ}}{=} \mathbb{P}\left(Y > 1 - \frac{1}{2}\right) = e^{-2 \cdot \frac{1}{2}} = e^{-1}$

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④ $X_i := \mathbb{1}[A_i]$, $A_i := \{$ i-EDIK GOLYÓ VALAHA KI LETT HÚZVA $\}$

$X = \sum_{i=1}^{45} X_i$ a) $\mathbb{E}(X) = \sum_{i=1}^{45} \mathbb{P}(A_i) = 45 \cdot \mathbb{P}(A_1)$

$\mathbb{P}(A_1) = 1 - \mathbb{P}($ SOSEM LETT KIHÚZVA AZ 1-ES GOLYÓ $) =$

$$= 1 - \left(\frac{\binom{44}{6}}{\binom{45}{6}}\right)^{20} = 1 - \left(\frac{39}{45}\right)^{20} =: p, \quad \mathbb{E}(X) = 45 \cdot p$$

b) $\mathrm{Var}(X) = \sum_{i,j=1}^{45} \mathrm{Cov}(X_i, X_j) =$

$$= 45 \cdot \underbrace{\mathrm{Var}(X_1)}_{= p\cdot(1-p)} + 45 \cdot 44 \cdot \underbrace{\mathrm{Cov}(X_1, X_2)}_{= \mathbb{P}(A_1 \cap A_2) - p^2}$$

$$\mathbb{P}(A_1 \cap A_2) \overset{\text{DE MORGAN}}{=} 1 - \mathbb{P}(A_1^c \cup A_2^c) \overset{\text{SZITA}}{=} 1 - \underbrace{\mathbb{P}(A_1^c)}_{1-p} - \underbrace{\mathbb{P}(A_2^c)}_{1-p} + \mathbb{P}(A_1^c \cap A_2^c)$$

$$\mathbb{P}(A_1^c \cap A_2^c) = \left(\frac{\binom{43}{6}}{\binom{45}{6}}\right)^{20} = \left(\frac{39 \cdot 38}{45 \cdot 44}\right)^{20}$$