VAL. SZÁM 1. PÓTZH 2, 2021.12.10.

① X := NARANCS TÖMEG, Y := MANDARIN TÖMEG

$X \sim N(2, (\frac{1}{5})^2)$, $Y \sim N(2, (\frac{1}{5})^2)$, F.A.E.

$500 \cdot X \sim N(1000, (\frac{500}{5})^2)$, $600 \cdot Y \sim N(1200, (\frac{600}{5})^2)$

$Z := 500 \cdot X + 600 \cdot Y$ = ENNYIT FIZETEK ÖSSZESEN

$Z \sim N(2200, 100^2 + 120^2)$ $\sqrt{100^2+120^2} = 156.2$

$$P(Z < 2000) = P\left(\frac{Z-2200}{156.2} < \frac{2000-2200}{156.2}\right) =$$

$$\Phi\left(-\frac{200}{156.2}\right) = 1 - \Phi(1.28) = 1 - 0.8997 = 0.1$$

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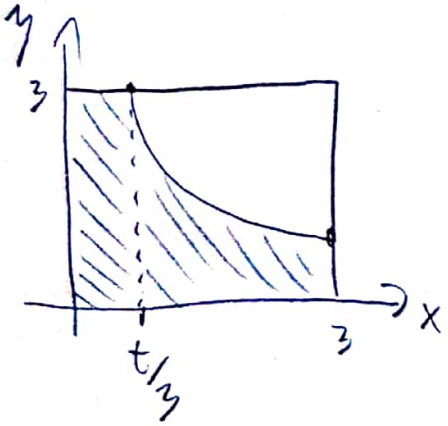
② $F(t) = P(T < t) = P(XY < t) = \begin{cases} 0, & \text{HA } t \le 0 \\ ?, & \text{HA } t \in (0,9) \\ 1, & \text{HA } t \ge 9 \end{cases}$

$? = P(Y < t/X) = \frac{\text{SATÍROZOTT SÍKIDOM TERÜLETE}}{3 \cdot 3} =$



$$= \frac{1}{9} \cdot \left(\frac{t}{3} \cdot 3 + \int_{t/3}^{3} \frac{t}{x}\,dx\right) =$$

$$= \frac{1}{9} \cdot \left(t + t \cdot (\ln(3) - \ln(t/3))\right)$$

1. OLDAL

(3) $A_i := \{ i\text{-EDIK MATEMATIKUSNAK CSAK MAT. CSAPATTÁRSA VAN} \}$, $i = 1, 2, \dots, 11$

$X_i := \mathbb{1}[A_i]$ $\quad X = X_1 + \dots + X_{11}$

$$\mathbb{E}(X) = \sum_{i=1}^{11} \mathbb{E}(X_i) = \sum_{i=1}^{11} \mathbb{P}(A_i) = 11 \cdot \frac{10 \cdot 9}{17 \cdot 16} = 3.64$$

$$\mathbb{P}(A_i) = \frac{\binom{10}{2}}{\binom{17}{2}} = \frac{10 \cdot 9}{17 \cdot 16}$$

BÓNUSZ: SZÓRÁS: $\sqrt{\mathrm{Var}(X)}$

$$\mathrm{Var}(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2 = \mathbb{E}(X^2) - \left(11 \cdot \frac{10 \cdot 9}{17 \cdot 16}\right)^2$$

$$\mathbb{E}(X^2) = \sum_{i,j=1}^{11} \mathbb{E}(X_i X_j) = \sum_{i,j=1}^{11} \mathbb{P}(A_i \cap A_j) = \text{(A)}$$

$i = j$: $\mathbb{P}(A_i \cap A_i) = \mathbb{P}(A_i) = \frac{10 \cdot 9}{17 \cdot 16} = p$

$i \neq j$: $B_{i,j} := \{ i \text{ ÉS } j \text{ EGY CSAPATBAN VANNAK} \}$

$$\mathbb{P}(A_i \cap A_j) = \underbrace{\mathbb{P}(A_i \cap A_j \cap B_{i,j})}_{= \frac{9}{\binom{17}{2}}} + \underbrace{\mathbb{P}(A_i \cap A_j \cap B_{i,j}^c)}_{= \frac{\binom{9}{2}}{\binom{17}{2}} \cdot \frac{\binom{7}{2}}{\binom{14}{2}}} = q$$

$$\text{(A)} = 11 \cdot p + 11 \cdot 10 \cdot q$$

2. OLDAL