**ELM 1** c) $E := \{$A KÉT KIHÚZOTT KETYERE HIBÁTLAN$\}$

$F_k := \{$A DOBOZ $k$ DARAB HIBÁS KETYERÉT TARTALMAZ$\}$

$$P(F_0|E) \overset{\text{BAYES}}{=} \frac{P(E|F_0)\cdot P(F_0)}{\sum_{k=0}^{2} P(E|F_k)\cdot P(F_k)} = \frac{1\cdot 1/2}{1\cdot 1/2 + \frac{6}{10}\cdot\frac{1}{4} + \frac{3}{10}\cdot\frac{1}{4}} =$$

$$\boxed{= \frac{20}{29} \approx 0.67}$$

HISZEN

$$P(E|F_k) = \frac{\binom{5-k}{2}}{\binom{5}{2}}, \text{ ÉS ÍGY}$$

$$P(E|F_0) = 1, \quad P(E|F_1) = \frac{\binom{4}{2}}{\binom{5}{2}} = \frac{6}{10}, \quad P(E|F_2) = \frac{\binom{3}{2}}{\binom{5}{2}} = \frac{3}{10}$$

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**ELM 2** a) $m := \mathbb{E}(X)$

DEF 1: $Var(X) = \mathbb{E}((X-m)^2)$

DEF 2: $Var(X) = \mathbb{E}(X^2) - m^2$

$$\mathbb{E}((X-m)^2) = \mathbb{E}(X^2 - 2mX + m^2) =$$

$$\mathbb{E}(X^2) - 2\cdot m\cdot \underbrace{\mathbb{E}(X)}_{m} + m^2 = \mathbb{E}(X^2) - m^2$$

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c) LÁSD 2. OLDAL

$\boxed{\text{ELM 2}}$ c) $Y = X_1 + \ldots + X_6$, AHOL

$X_r :=$ AHÁNY KÖRT JÁTSZANAK ANNA $r-1$-EDIK GYŐZELME UTÁN A $r$-ADIK GYŐZELMÉIG

EKKOR $X_1, \ldots, X_6$ F.A.E. $GEO\left(\frac{1}{3}\right)$

(AZAZ $Y \sim$ NEGATÍV BINOMIÁLIS $\left(6, \frac{1}{3}\right)$)

$$\mathbb{E}(X_r) = \frac{1}{1/3} = 3 \qquad Var(X_r) = \frac{1-\frac{1}{3}}{(1/3)^2} = 6$$

$$\mathbb{E}(Y) = 6 \cdot \mathbb{E}(X_r) = 18 \qquad Var(Y) = 6 \cdot Var(X_r) = 36$$

$$\mathbb{P}(Y \in (9, 27)) = \mathbb{P}(|Y - 18| < 9) =$$

$$1 - \underbrace{\mathbb{P}(|Y - 18| \geq 9)}_{\leq \frac{Var(Y)}{9^2} = \frac{36}{81} = \frac{4}{9}} \geq 1 - \frac{4}{9} = \frac{5}{9}$$

CSEB

HISZ $\mathbb{E}(Y) = 18$

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$\boxed{\text{2. OLDAL}}$

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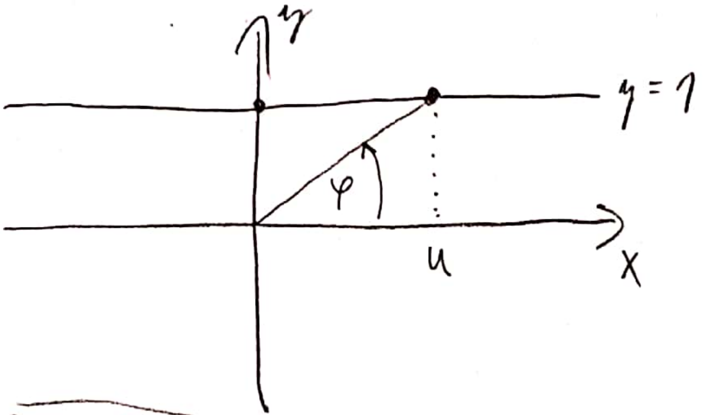
$\boxed{\text{ELM 3}}$ b) SZIMMETRIA MIATT FELTEHETJÜK, HOGY A FELSŐ FALAT TALÁLJA EL A LÖVEDÉK, AZAZ $\varphi \sim UNI[0,\pi]$




$$\frac{\sin(\varphi)}{\cos(\varphi)} = \frac{1}{u} \Rightarrow$$

$$u = \mathrm{ctg}(\varphi) = \tan\left(\frac{\pi}{2} - \varphi\right)$$

$$\boxed{\alpha := \frac{\pi}{2} - \varphi} \Rightarrow \alpha \sim UNI\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \quad u = \tan(\alpha)$$

$$F(x) = \mathbb{P}(u \le x) = \mathbb{P}(\tan(\alpha) \le x) = \mathbb{P}(\alpha \le \arctan(x)) =$$

$$= \frac{\arctan(x) + \frac{\pi}{2}}{\pi} \qquad f(x) = F'(x) = \frac{1}{\pi} \cdot \frac{1}{1+x^2}$$

(CAUCHY ELOSZLÁS)

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$\boxed{\text{GYAK 1}}$ JELENTKEZŐK SZÁMA: $BIN\left(\frac{3}{2} \cdot 10^5, 10^{-4}\right)$

POISSON KÖZELÍTÉSSEL: $POI(15)$

a) FELVETTEK SZÁMA: $BIN\left(\frac{3}{2} \cdot 10^5, 10^{-4} \cdot \frac{1}{3} \cdot \frac{1}{5}\right) \sim POI(1)$

$\mathbb{P}(\text{LEGF. EGYET VESZNEK FEL}) = e^{-1} + e^{-1} \cdot 1 = 2 \cdot e^{-1}$

b) $BIN\left(6, \frac{1}{5}\right)$ $\mathbb{P}(\text{LEGALÁBB KETTŐT}) =$

$1 - \mathbb{P}(\text{LEGF. EGYET}) = 1 - \left(\frac{4}{5}\right)^6 - 6 \cdot \left(\frac{4}{5}\right)^5 \cdot \frac{1}{5}$

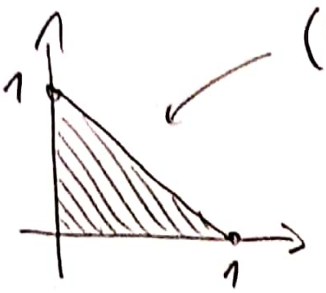
c) POISSON SZÍNEZÉSE MIATT A BUKOTT ÉS FELVETT JELENTKEZŐK SZÁMA FÜGGETLENNEK TEKINTHETŐ.

ÍGY A VÁLASZ $e^{-1} \cdot \frac{1^2}{2!} = \frac{1}{2} e^{-1}$

GYAK 2  LEGYEN $\boxed{U := -\mathbb{X}}$ $\boxed{V := -\mathbb{Y}}$

$$\mathrm{Var}(U) = \mathrm{Var}(\mathbb{X}), \quad \mathrm{Cov}(U,V) \overset{\text{BIL.}}{=} (-1)\cdot(-1)\cdot \mathrm{Cov}(\mathbb{X},\mathbb{Y}) = \mathrm{Cov}(\mathbb{X},\mathbb{Y})$$

ÍGY ELÉG KISZÁMOLNI $(U,V)$ KOVARIANCIA-MÁTRIXÁT.



$(U,V)$ EZEN A HÁROMSZÖGÖN EGYENLETES

$$f(u,v) = 2 \cdot \mathbb{1}[0 \le u,\ 0 \le v,\ v \le 1-u]$$

$$f_U(u) = 2\cdot(1-u)\cdot \mathbb{1}[0 \le u \le 1]$$

$$\mathbb{E}(U) = \int_0^1 f_U(u)\cdot u\,du = \int_0^1 2\cdot(1-u)\cdot u\,du = 2\cdot\int_0^1 (u-u^2)\,du = \frac{1}{3}$$

$$\mathbb{E}(U^2) = \int_0^1 f_U(u)\cdot u^2\,du = \int_0^1 2\cdot(1-u)\cdot u^2\,du = 2\cdot\int_0^1 (u^2-u^3)\,du = \frac{1}{6}$$

$$\mathrm{Var}(U) = \frac{1}{6} - \left(\frac{1}{3}\right)^2 = \frac{1}{18} \overset{\text{SZIMM.}}{=\!=} \mathrm{Var}(V)$$

$$\mathbb{E}(U\cdot V) = \int_0^1\int_0^{1-u} f(u,v)\cdot u\cdot v\,dv\,du = \int_0^1\int_0^{1-u} 2\cdot u\cdot v\,dv\,du =$$

$$= 2\cdot\int_0^1 u\cdot\int_0^{1-u} v\,dv\,du = \int_0^1 u\cdot(1-u)^2\,du = \int_0^1 (u-2u^2+u^3)\,du =$$

$$= \frac{1}{2} - 2\cdot\frac{1}{3} + \frac{1}{4} = \frac{1}{12} \qquad \mathrm{Cov}(U,V) = \frac{1}{12} - \frac{1}{3}\cdot\frac{1}{3} = -\frac{1}{36}$$

KOVARIANCIA-MÁTRIX:

| | |
|---|---|
| $\mathrm{Var}(U)$ | $\mathrm{Cov}(U,V)$ |
| $\mathrm{Cov}(V,U)$ | $\mathrm{Var}(V)$ |

=

| | |
|---|---|
| $\frac{1}{18}$ | $-\frac{1}{36}$ |
| $-\frac{1}{36}$ | $\frac{1}{18}$ |

GYAK 3 $\mathcal{X}_i$ := XAVÉR i-EDIK KOCKADOBÁSÁNAK ERED.

$\mathcal{Y}_i$ := KVETT —"— —"— —"—

$$S^{\mathcal{X}} := \mathcal{X}_1 + \dots + \mathcal{X}_{9\cdot 10^6}, \quad S^{\mathcal{Y}} := \mathcal{Y}_1 + \dots + \mathcal{Y}_{16\cdot 10^6}$$

$$E(\mathcal{X}_i) = E(\mathcal{Y}_i) = 3.5 \quad Var(\mathcal{X}_i) = Var(\mathcal{Y}_i) = \frac{91}{6} - (3.5)^2 = \frac{35}{12}$$

$$\sigma := \sqrt{\frac{35}{12}} \quad \mathcal{X}^* := \frac{S^{\mathcal{X}} - 3.5\cdot 9\cdot 10^6}{\sigma\cdot 3\cdot 10^3}, \quad \mathcal{Y}^* := \frac{S^{\mathcal{Y}} - 3.5\cdot 16\cdot 10^6}{\sigma\cdot 4\cdot 10^3}$$

$$\text{C.H.T.} \Rightarrow \mathcal{X}^* \approx \mathcal{N}(0,1), \quad \mathcal{Y}^* \approx \mathcal{N}(0,1), \quad \mathcal{X}^*, \mathcal{Y}^* \text{ F.A.E.}$$

$$\mathcal{X} = S^{\mathcal{X}}/(9\cdot 10^6), \quad \mathcal{Y} = S^{\mathcal{Y}}/(16\cdot 10^6)$$

$$\mathcal{X} - 3.5 = \frac{S^{\mathcal{X}} - 3.5\cdot 9\cdot 10^6}{9\cdot 10^6} = \frac{\mathcal{X}^*\cdot\sigma\cdot 3\cdot 10^3}{9\cdot 10^6} = \frac{\mathcal{X}^*\cdot\sigma}{3\cdot 10^3}$$

$$\mathcal{Y} - 3.5 = \dots = \frac{\mathcal{Y}^*\cdot\sigma}{4\cdot 10^3}$$

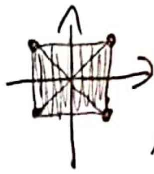
a)

$$P\left(|\mathcal{X} - 3.5| \le 10^{-3}, |\mathcal{Y} - 3.5| \le \frac{3}{4}\cdot 10^{-3}, |\mathcal{X} - 3.5| \ge \frac{4}{3}\cdot(\mathcal{Y} - 3.5)\right) =$$

$$P\left(\left|\frac{\mathcal{X}^*\cdot\sigma}{3\cdot 10^3}\right| \le 10^{-3}, \left|\frac{\mathcal{Y}^*\cdot\sigma}{4\cdot 10^3}\right| \le \frac{3}{4}\cdot 10^{-3}, \left|\frac{\mathcal{X}^*\cdot\sigma}{3\cdot 10^3}\right| \ge \frac{4}{3}\cdot\frac{\mathcal{Y}^*\cdot\sigma}{4\cdot 10^3}\right) =$$

$$P\left(|\mathcal{X}^*| \le \frac{3}{\sigma}, |\mathcal{Y}^*| \le \frac{3}{\sigma}, |\mathcal{X}^*| \ge \mathcal{Y}^*\right) = ☺$$

LÁSD 6. OLDAL

$☺ = \mathbb{P}\left( (X^*, Y^*) \in$  $\right) = \frac{3}{4} \cdot \mathbb{P}\left( (X^*, Y^*) \in$  $\right)$

2-DIM STANDARD NORMÁLIS FORGÁS-INVARIANCIÁJA

$$= \frac{3}{4} \cdot \mathbb{P}\left( |X^*| \le \frac{3}{\sigma} \right) \cdot \mathbb{P}\left( |Y^*| \le \frac{3}{\sigma} \right) = \frac{3}{4} \cdot \left( 2 \cdot \Phi\left(\frac{3}{\sigma}\right) - 1 \right)^2$$

---

b) $\mathbb{P}\left( 9 \cdot (X - 3.5)^2 + 16 \cdot (Y - 3.5)^2 \le 4 \cdot 10^{-6} \right) =$

$$\mathbb{P}\left( 9 \cdot \left( \frac{X^* \cdot \sigma}{3 \cdot 10^3} \right)^2 + 16 \cdot \left( \frac{Y^* \cdot \sigma}{4 \cdot 10^3} \right)^2 \le 4 \cdot 10^{-6} \right) =$$

POLÁR

$$\mathbb{P}\left( (X^*)^2 + (Y^*)^2 \le \frac{4}{\sigma^2} \right) = \mathbb{P}\left( \|(X^*, Y^*)\| \le \frac{2}{\sigma} \right) =$$

$$\int_0^{2\pi} \int_0^{2/\sigma} \frac{1}{2\pi} \cdot e^{-r^2/2} \cdot r \, dr \, d\varphi = \left[ -e^{-r^2/2} \right]_0^{2/\sigma} = 1 - e^{-2/\sigma^2}$$

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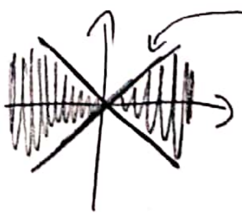
BÓNUSZ: KELL: $\mathbb{P}\left( |Z - 3.5| < |X - 3.5| \right) = \frac{1}{3}$

ZEBULON DOBÁSAINAK SZÁMA: $\beta \cdot 10^6$

$$\frac{1}{3} = \mathbb{P}\left( |Z - 3.5| < |X - 3.5| \right) = \mathbb{P}\left( \left| \frac{Z^* \cdot \sigma}{\sqrt{\beta} \cdot 10^3} \right| < \left| \frac{X^* \cdot \sigma}{3 \cdot 10^3} \right| \right) =$$

MEREDEKSÉG: $\sqrt{\beta}/3$

$\mathbb{P}\left( |Z_1^*| < \frac{\sqrt{\beta}}{3} \cdot |X^*| \right) = \mathbb{P}\left( (X^*, Z_1^*) \in \right.$  $\left. \right) =$

$$= \frac{4\alpha}{2\pi} = \frac{1}{3} \Rightarrow \boxed{\alpha = \frac{\pi}{6}} \Rightarrow \frac{\sqrt{\beta}}{3} = \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} \Rightarrow$$

$\Rightarrow \boxed{\beta = 3} \Rightarrow$ ZEBULON $3 \cdot 10^6$-SZOR DOBJA FEL

2 DIM STANDARD NORMÁLIS SZÖGE: UNI$[0, 2\pi]$