VSZ1 ZH2, 2023.11.23., A CSOPORT

① $\mathcal{X}$ = NAPSZÚRÁSOS GYEREKEK SZÁMA

$\mathcal{X} \sim BIN\left(1000, \frac{1}{4}\right)$ KELL $k \in \mathbb{N}$, HOGY

$\mathbb{P}(\mathcal{X} \leq k) \stackrel{?}{=} 0.99$

DE MOIVRE

$$\mathbb{P}(\mathcal{X} \leq k) = \mathbb{P}\left(\frac{\mathcal{X} - 250}{\sqrt{10^3 \cdot \frac{1}{4} \cdot \frac{3}{4}}} \leq \frac{k - 250}{\sqrt{10^3 \cdot \frac{1}{4} \cdot \frac{3}{4}}}\right) \approx$$

$$\Phi\left(\frac{k-250}{13.7}\right) = 0.99 \xrightarrow[\text{LÁZAT}]{\text{TÁB-}} \frac{k-250}{13.7} = 2.33$$

$$\Longrightarrow k = \lceil 281.9 \rceil = 282 \text{ TABLETTA KELL}$$

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② a) $F(x) = \mathbb{P}(\xi \leq x) = 1 - e^{-x}$ (HA $x > 0$)

$$G(x) = \mathbb{P}(\mathcal{X} \leq x) = \mathbb{P}(\sqrt{\xi} \leq x) \stackrel{\downarrow}{=} \mathbb{P}(\xi \leq x^2) =$$

$$= 1 - e^{-x^2} \text{ (HA } x > 0)$$

$$g(x) = G'(x) = 2x \cdot e^{-x^2} \cdot \mathbb{1}[x > 0]$$

b) $\mathbb{P}(\mathcal{X} > 2 \mid \mathcal{X} > 1) = \mathbb{P}(\sqrt{\xi} > 2 \mid \sqrt{\xi} > 1) =$

$$\mathbb{P}(\xi > 4 \mid \xi > 1) \xrightarrow[\text{IFJÚ}]{\text{ÖRÖK}} \mathbb{P}(\xi > 3) = e^{-3}$$

BÓNUSZ: $\mathbb{E}(\mathcal{X}) = \int_0^\infty x \cdot g(x)\,dx = \int_0^\infty 2x^2 \cdot e^{-x^2}\,dx =$ ☺

1. OLDAL

$$☺ = \int_{-\infty}^{\infty} x^2 \cdot e^{-x^2} \, dx = \int_{-\infty}^{\infty} x^2 \cdot \exp\left(-\frac{x^2}{2 \cdot \sigma^2}\right) dx = ☺$$

(AHOL $\sigma = \frac{1}{\sqrt{2}}$)

$$☺ = \sqrt{2\pi} \cdot \sigma \cdot \underbrace{\int_{-\infty}^{\infty} x^2 \cdot \frac{1}{\sqrt{2\pi} \cdot \sigma} \cdot \exp\left(-\frac{x^2}{2\sigma^2}\right) dx}_{\mathrm{Var}(Y) = \sigma^2 = \frac{1}{2}}$$

$\boxed{\text{AHOL } Y \sim \mathcal{N}(0, \sigma^2)}$

$$\mathrm{Var}(Y) = \sigma^2 = \frac{1}{2}$$

ÍGY $☺ = \sqrt{\pi}/2 \approx 0.886$

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③ X = SZORONGÓ GYEREKEK SZÁMA

$X_i := \mathbb{1}[A_i]$, $A_i := \{$i-EDIK SZÉKEN ÜLŐ GYEREK SZORONG$\}$

$X = \sum_{i=1}^{40} X_i$, $\mathbb{E}(X) = \sum_{i=1}^{40} \mathbb{P}(A_i) = @$

NÉGY GYEREK ÜL A SOR SZÉLÉN.

SOR SZÉLE: $\mathbb{P}(A_i) = \frac{20}{39}$

SOR KÖZEPE: $\mathbb{P}(A_i) = 1 - \frac{19}{39} \cdot \frac{18}{38}$

$$@ = 4 \cdot \frac{20}{39} + 36 \cdot \left(1 - \frac{19}{39} \cdot \frac{18}{38}\right) = \frac{1160}{39} = 29.74$$

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(1) $X \sim N(9, 2^2)$, $Y \sim N(8, 2^2)$, $Z \sim N(7, 1^2)$

$\boxed{W := X + Y + Z}$ $W \sim N(\mu, \sigma^2)$, AHOL

$\mu = 9 + 8 + 7 = 24$ $\sigma^2 = 4 + 4 + 1 = 9 \Rightarrow \sigma = 3$

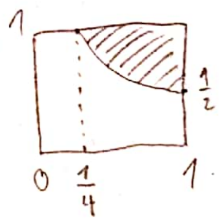
$\mathbb{P}$(NEM HALADJA MEG) $= \mathbb{P}(W \leq 25) =$

$\mathbb{P}\left(\frac{W-24}{3} \leq \frac{25-24}{3}\right) = \Phi\left(\frac{1}{3}\right)$ TÁBLÁZAT

0.6293

(2) X, Y F.A.E. UNI$[0,1]$

$V = X \cdot Y^2 \cdot \pi$ $\mathbb{P}\left(V > \frac{\pi}{4}\right) = \mathbb{P}\left(X \cdot Y^2 \cdot \pi > \frac{\pi}{4}\right) =$

$= \mathbb{P}\left(Y > \frac{1}{2} \cdot \frac{1}{\sqrt{X}}\right) =$ TERÜLET () $=$

$= \int_{1/4}^{1} \left(1 - \frac{1}{2}\frac{1}{\sqrt{x}}\right) dx = \frac{3}{4} - \int_{1/4}^{1} \frac{1}{2} \cdot \frac{1}{\sqrt{x}} dx =$

$= \frac{3}{4} - \left[\sqrt{x}\right]_{1/4}^{1} = \frac{3}{4} - \left(\sqrt{1} - \sqrt{\frac{1}{4}}\right) = \frac{1}{4}$

(3) $Z_i := \mathbb{1}[A_i]$, $A_i :=$ {i-EDIK UTAS EGYEDÜL UTAZIK}

$Z = Z_1 + \ldots + Z_6$ LAPOZZ!

3. OLDAL

$$\mathbb{E}(Z_1) = \mathbb{E}(Z_{11}) + \ldots + \mathbb{E}(Z_{16}) = 6 \cdot \mathbb{P}(A_1)$$

$\mathbb{P}(A_1) = \mathbb{P}($A TÖBBI UTAS NEM ARRA A BUSZRA ÜL, MINT AZ ELSŐ UTAS$) =$

$= \left(\frac{7}{8}\right)^5$, így $\mathbb{E}(Z_1) = 6 \cdot \left(\frac{7}{8}\right)^5 = 3{,}077$

<u>BÓNUSZ:</u> $\mathrm{Var}(Z_1) = \mathbb{E}(Z_1^2) - \mathbb{E}(Z)^2$

$$\mathbb{E}(Z_1^2) = \mathbb{E}\left(\left(\sum_{i=1}^{6} Z_i\right) \cdot \left(\sum_{i=1}^{6} Z_j\right)\right) = \sum_{i,j=1}^{6} \mathbb{E}(Z_i Z_j) =$$

$$= \sum_{i,j=1}^{6} \mathbb{P}(A_i \cap A_j) = 6 \cdot \underbrace{\mathbb{P}(A_1)}_{\left(\frac{7}{8}\right)^5} + 30 \cdot \mathbb{P}(A_1 \cap A_2)$$

$$\mathbb{P}(A_1 \cap A_2) = \underbrace{\mathbb{P}(A_1)}_{\left(\frac{7}{8}\right)^5} \cdot \underbrace{\mathbb{P}(A_2 | A_1)}_{\left(\frac{6}{7}\right)^4}$$

$$\mathrm{Var}(Z_1) = 6 \cdot \left(\frac{7}{8}\right)^5 + 30 \cdot \left(\frac{7}{8}\right)^5 \cdot \left(\frac{6}{7}\right)^4 - \left(6 \cdot \left(\frac{7}{8}\right)^5\right)^2 =$$

$$= 1{,}9123$$