VSZ1 ZH2, 2022.11.24, A CSOP.

(1) $X :=$ RELEVÁNS ÁLLOMÁNYOK SZÁMA

$X \sim BIN(900, 0.8)$ KELL: $k \in \mathbb{N}$, HOGY

$\mathbb{P}(X \le k) = 0.98$. AZAZ: $\mathbb{P}(X \le k) =$ DE MOIVRE

$$\mathbb{P}\left(\frac{X - 900\cdot 0.8}{\sqrt{900\cdot 0.8\cdot 02}} \le \frac{k - 900\cdot 0.8}{\sqrt{900\cdot 0.8\cdot 0.2}}\right) \approx \Phi\left(\frac{k-900\cdot 0.8}{\sqrt{900\cdot 0.8\cdot 0.2}}\right) = 0.98$$

AZAZ A TÁBLÁZAT ALAPJÁN:

$$\frac{k - 900\cdot 0.8}{\sqrt{900\cdot 08\cdot 02}} = 2.06 \Longrightarrow k = \lceil 744.72 \rceil = 745$$

TEHÁT A HÁTTÉRTÁR MÉRETE LEGYEN LEGALÁBB $745 \cdot 10$ MByte, AZAZ $\boxed{7450 \text{ MByte}}$ 6

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(2) $A_i := \{$ i-EDIK DOBÁS HARMONIKUS $\}$, $i = 2, \dots, 49$

$B_i := \{ \xi_{i-1} + \xi_{i+1}$ PÁROS SZÁM $\}$

$$\mathbb{P}(A_i) = \mathbb{P}(A_i \cap B_i) = \underbrace{\mathbb{P}(A_i \mid B_i)}_{1/6} \cdot \underbrace{\mathbb{P}(B_i)}_{1/2} = \frac{1}{12}$$

$$X = \sum_{i=2}^{49} \mathbb{1}[A_i], \text{ ÍGY } \mathbb{E}(X) \overset{LIN}{=} \sum_{i=2}^{49} \mathbb{E}(\mathbb{1}[A_i]) =$$

$$= \sum_{i=2}^{49} \mathbb{P}(A_i) = 48 \cdot \frac{1}{12} = 4$$

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1. OLDAL

3) a) $f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_{0}^{\pi/2} A\cdot \sin(x+y)\,dy =$

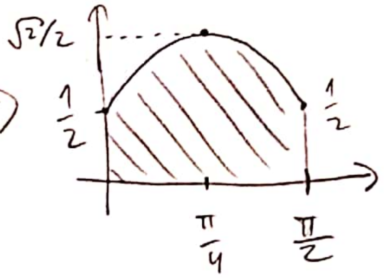
$= A\cdot\left[-\cos(x+y)\right]_{y=0}^{y=\pi/2} = A\cdot\left(\cos(x)-\cos\left(x+\pi/2\right)\right)$, HA $0<x<\frac{\pi}{2}$

$1 = \int_{-\infty}^{\infty} f_X(x)\,dx = \int_{0}^{\pi/2} A\cdot\left(\cos(x)-\cos\left(x+\pi/2\right)\right)dx =$

$= A\cdot\left[\sin(x)-\sin\left(x+\frac{\pi}{2}\right)\right]_{x=0}^{x=\frac{\pi}{2}} =$

$= A\cdot\left(\sin\left(\frac{\pi}{2}\right)-\sin(\pi)-\sin(0)+\sin\left(\frac{\pi}{2}\right)\right) = A\cdot 2 \Rightarrow \boxed{A=\frac{1}{2}}$

ÍGY $f_X(x) = \frac{1}{2}\cdot\left(\cos(x)-\cos\left(x+\frac{\pi}{2}\right)\right) = \frac{1}{2}\cdot\left(\cos(x)+\sin(x)\right)$
HA $0<x<\frac{\pi}{2}$



$f(x,y)=f(y,x)$, ÍGY SZIMMETRIA MIATT

$f_Y(y) \equiv f_X(y) = \frac{1}{2}\cdot\left(\cos(y)+\sin(y)\right)$
HA $0<x<\frac{\pi}{2}$

4

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b) $f_{Y|X}\left(y\,|\,\frac{\pi}{4}\right) = \frac{f\left(\frac{\pi}{4},y\right)}{f_X\left(\frac{\pi}{4}\right)} = \frac{\frac{1}{2}\cdot\sin\left(\frac{\pi}{4}+y\right)}{\frac{1}{2}\cdot\left(\cos\left(\frac{\pi}{4}\right)+\sin\left(\frac{\pi}{4}\right)\right)} = \frac{\sin\left(\frac{\pi}{4}+y\right)}{\sqrt{2}}$
HA $0<y<\frac{\pi}{2}$

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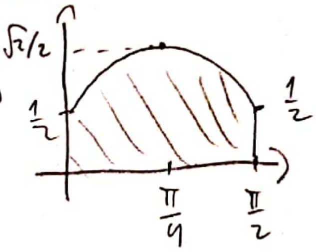
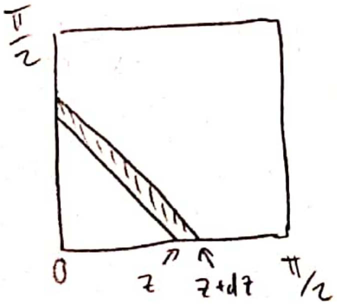


$\mathbb{P}\left(Y<\frac{\pi}{4}\,\middle|\,X=\frac{\pi}{4}\right) = \frac{1}{2}$ SZIMM. MIATT.

BÓNUSZ:



$f_{Z_1}(z) = \frac{1}{2}\cdot\left(z\wedge(\pi-z)\right)\cdot\sin(z)$

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2. OLDAL