VALÓSZÁM 1., ZH2 PÓT, 2023.12.08.

① X = EGY NAP SEJEJTETETT TV-K SZÁMA

$X \sim BIN(100, 0.2)$ $\{SZOM\} = \{X \geq 25\}$

a) $\mathbb{P}(SZOM) = \mathbb{P}(X \geq 25) = \mathbb{P}\left(\frac{X - \mathbb{E}(X)}{\sqrt{Var(X)}} \geq \frac{25 - \mathbb{E}(X)}{\sqrt{Var(X)}}\right) =$

$= \mathbb{P}\left(\frac{X - 100 \cdot 0.2}{\sqrt{100 \cdot 02 \cdot 0.8}} \geq \frac{25-20}{4}\right) \overset{\text{DE MOIVRE}}{\approx} \mathbb{P}\left(\underset{\sim N(0,1)}{Y} \geq \frac{25-20}{4}\right) =$

$= 1 - \Phi\left(\frac{25-20}{4}\right) = 1 - \Phi\left(\frac{5}{4}\right) = 1 - 0.8944 = 0.1056$

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b) Z_1 = SZOMORÚ NAPOK SZÁMA JÖVŐRE

$Z_1 \sim BIN(365, 0.1056)$ $\mathbb{P}(Z_1 \leq 33) =$

$= \mathbb{P}\left(\frac{Z_1 - \mathbb{E}(Z_1)}{\sqrt{Var(Z_1)}} \leq \frac{33 - \mathbb{E}(Z_1)}{\sqrt{Var(Z_1)}}\right) \overset{\text{DE MOIVRE}}{\approx} \Phi\left(\frac{33 - 365 \cdot 0.1056}{\sqrt{365 \cdot 0.1056 \cdot (1 - 0.1056)}}\right)$

$= \Phi(-0.944) = 1 - \Phi(0.944) = 1 - 0.8264 = 0.1736$

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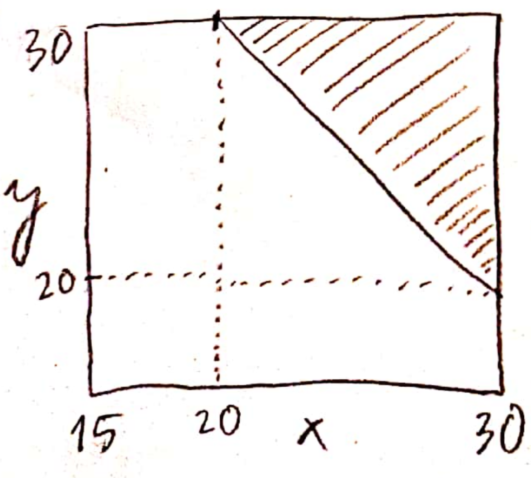
②



$\mathbb{P}(8.00 \text{ ELŐTT ÉREK BE}) =$

$\mathbb{P}(X + Y \leq 50) = 1 - \mathbb{P}(X + Y > 50)$

$= 1 - \frac{\text{SATÍROZOTT TERÜLET}}{\text{ÖSSZ - TERÜLET}} =$

$= 1 - \frac{10^2/2}{15^2} = \frac{7}{9} = 0.7777$

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③ $\mathfrak{X}$ = IRIGY KISGYEREKEK SZÁMA

$\mathfrak{X} = \mathfrak{X}_1 + \ldots + \mathfrak{X}_{14}$, $\mathfrak{X}_i = \mathbb{1}[A_i]$, $A_i = \{$ i-EDIK GYEREK IRIGY $\}$

$$\mathbb{E}(\mathfrak{X}) = \mathbb{E}(\mathfrak{X}_1) + \ldots + \mathbb{E}(\mathfrak{X}_{14}) = 14 \cdot \mathbb{E}(\mathfrak{X}_1) = 14 \cdot \mathbb{P}(A_1)$$

$$\mathbb{P}(A_1) = \underbrace{\mathbb{P}(\text{NEKI KESŐ})}_{\frac{7}{14} = \frac{1}{2}} \cdot \underbrace{\mathbb{P}(\text{SZEMBE TEJ} \mid \text{NEKI KESERŰ})}_{\frac{7}{13}}$$

ÍGY $\mathbb{P}(A_1) = \frac{7}{26}$

ÍGY $\mathbb{E}(\mathfrak{X}) = 14 \cdot \frac{7}{26} = 3.7692$

<u>BÓNUSZ</u>: $\mathrm{Var}(\mathfrak{X}) = \mathbb{E}(\mathfrak{X}^2) - \left(14 \cdot \frac{7}{26}\right)^2 = ☆$

$$\mathbb{E}(\mathfrak{X}^2) = \mathbb{E}\left(\sum_{i,j=1}^{14} \mathfrak{X}_i \mathfrak{X}_j\right) = \sum_{i,j=1}^{14} \mathbb{P}(A_i \cap A_j) = ☺$$

$\boxed{i = j} \Rightarrow \mathbb{P}(A_i \cap A_j) = \mathbb{P}(A_1) = \frac{7}{26}$

$\boxed{\text{i ÉS j SZEMBEN LAKNAK}} \Longrightarrow \mathbb{P}(A_i \cap A_j) = 0$

$\boxed{\text{i} \neq \text{j ÉS NEM SZEMBEN LAKNAK}}$:

$$\mathbb{P}(A_i \cap A_j) = \underbrace{\mathbb{P}(A_i)}_{7/26} \cdot \underbrace{\mathbb{P}(A_j \mid A_i)}_{\frac{6}{12} \cdot \frac{6}{11} = \frac{6}{22}} = \frac{7}{26} \cdot \frac{6}{22}$$

$$☺ = 14 \cdot \frac{7}{26} + 14 \cdot 0 + \left((14)^2 - 28\right) \cdot \frac{7}{26} \cdot \frac{6}{22} = 16.1049$$

$$☆ = 16.1049 - \left(14 \cdot \frac{7}{26}\right)^2 = 1.8978$$