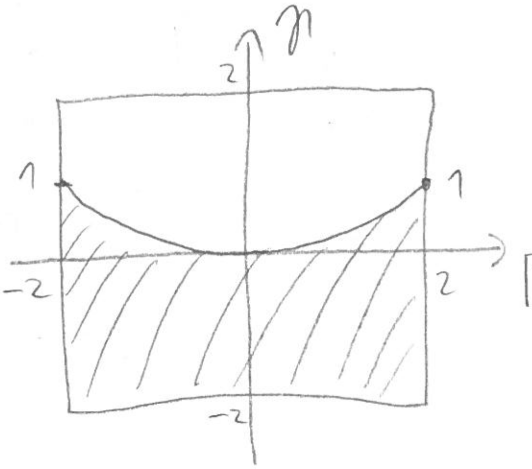
VSZ1, 2017, NOV 23, ZH2.

1) $\mathbb{P}$( DISZKRIMINÁNS POZITÍV) =

$$\mathbb{P}(\beta^2 - 4\gamma > 0) = \mathbb{P}\left(\left(\tfrac{\beta}{2}\right)^2 > \gamma\right) =$$




$$\frac{\text{KEDVEZŐ TERÜLET}}{4 \cdot 4} =$$

$$= \frac{1}{16} \cdot \int_{-2}^{2} \left(2 + \left(\frac{x}{2}\right)^2\right) dx = \frac{7}{12}$$

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2) EZER FORINT A MÉRTÉKEGYSÉG

$$X = \text{BEFIZETÉS} \sim \mathcal{N}(\underbrace{4 \cdot 30}_{120}, \underbrace{4 \cdot 2^2}_{16})$$

$$Y = \text{KÖLTÉS} \sim \mathcal{N}(115, 3^2)$$

a) $\mathbb{P}(Y < X) = \mathbb{P}(Y - X < 0) = \star$

$(Y - X) \sim \mathcal{N}(-5, 25)$ $\quad \star = \mathbb{P}\left(\frac{Y - X + 5}{5} < \frac{5}{5}\right)$

$= \Phi(1) = 0.8413$

② b) $Y \sim BIN(n, p)$, $\boxed{n = 9 \cdot 10^4}$ $\boxed{p = 0.8413}$

$$\mathbb{P}(Y < k) = \mathbb{P}\left(\frac{Y - n \cdot p}{\sqrt{n \cdot p \cdot (1-p)}} < \frac{k - n \cdot p}{\sqrt{n \cdot p \cdot (1-p)}}\right) \approx$$

DE MOIVRE-LAPLACE

$$\approx \Phi\left(\frac{k - n \cdot p}{\sqrt{n \cdot p \cdot (1-p)}}\right) \overset{\text{KELL}}{=\!=} 0.8$$

$\Phi^{-1}(0.8) = 0.85$, TEHÁT KELL:

$$0.85 = \frac{k - n \cdot p}{\sqrt{n \cdot p \cdot (1-p)}} = \frac{k - 75717}{110} \Rightarrow \boxed{k = 75810}$$

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③ $f_X(x) = \frac{1}{2} \cdot \mathbb{1}[3 < x < 5]$

$f_{Y|X}(y|x) = x \cdot e^{-x \cdot y} \cdot \mathbb{1}[y > 0]$

a) $f(x,y) = f_X(x) \cdot f_{Y|X}(y|x) =$

$$= \frac{1}{2} \cdot x \cdot e^{-xy} \cdot \mathbb{1}[y > 0, 3 < x < 5]$$

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b) $\mathbb{E}(Y | X = x_0) = \frac{1}{x_0}$, HISZEN EZ AZ $EXP(x_0)$ VÁRHATÓ ÉRTÉKE

c) $$\mathbb{E}(Y) = \int_{-\infty}^{\infty} \mathbb{E}(Y \mid X = x) \cdot f_X(x)\, dx \overset{(b)}{=}$$

$$= \int_3^5 \frac{1}{x} \cdot \frac{1}{2}\, dx = \frac{1}{2}\left(\ln(5) - \ln(3)\right)$$

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d) $$\mathrm{Cov}(X, Y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x \cdot y \cdot f(x,y)\, dy\, dx - \mathbb{E}(X)\cdot\mathbb{E}(Y)$$

$$= \underbrace{\int_3^5\int_0^{\infty} x \cdot y \cdot \frac{1}{2} x \cdot e^{-xy}\, dy\, dx}_{} - \left(\frac{3+5}{2}\right) \cdot \frac{1}{2}\left(\ln(5) - \ln(3)\right)$$

$$\hookrightarrow \int_3^5 \frac{1}{2} x \cdot \underbrace{\int_0^{\infty} y \cdot x \cdot e^{-xy}\, dy}_{= \frac{1}{x}\text{, HISZ EXP(x) VÁRHATÓ ÉRTÉKE}}\, dx = \int_3^5 \frac{1}{2} x \cdot \frac{1}{x}\, dx = 1$$

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4) $A_i := \{ i\text{-EDIK DOBÁS FEJ} \}$ $A_i^c := \{ i\text{-EDIK DOBÁS ÍRÁS} \}$

$B_i := A_i \cap A_{i+1} = \{ i+1\text{-KOR JOBBRA LÉP} \}$

$C_i := A_i^c \cap A_{i+1}^c = \{ i+1\text{-KOR BALRA LÉP} \}$

$X_i := \mathbb{1}[B_i]$, $Y_i := \mathbb{1}[C_i]$

$\xi_+ = \sum_{i=1}^{99} X_i$, $\xi_- = \sum_{i=1}^{99} Y_i$

$\mathbb{E}(\xi_+) = 99 \cdot \mathbb{P}(B_1) = 99 \cdot \left(\frac{3}{4}\right)^2$

$$\mathbb{E}(\xi_-) = 99\cdot\mathbb{P}(C_1) = 99\cdot\left(\frac{1}{4}\right)^2$$

$$\mathbb{E}(X) = \mathbb{E}(\xi_+ - \xi_-) = 99\cdot\frac{9}{16} - 99\cdot\frac{1}{16}$$

$$\mathrm{Cov}(\xi_+, \xi_-) = \mathrm{Cov}\left(\sum_{i=1}^{99} X_i, \sum_{j=1}^{99} Y_j\right) \overset{\text{BILINEARITÁS}}{=\!=}$$

$$= \sum_{i,j=1}^{99} \mathrm{Cov}(X_i, Y_j) = ☺$$

$$\boxed{|i-j| \geq 2} \Longrightarrow \mathrm{Cov}(X_i, Y_j) \overset{\text{FÜGGETLENSÉG}}{=} 0$$

$$\mathrm{Cov}(X_i, Y_j) = \underbrace{\mathbb{P}(B_i \cap C_j)}_{=0} - \left(\frac{3}{4}\right)^2\cdot\left(\frac{1}{4}\right)^2 = -\frac{9}{256}$$

$$\boxed{|i-j| \leq 1} \Longrightarrow \quad = 0$$ , TEHÁT

$$☺ = (99 + 2\cdot 98)\cdot\left(-\frac{9}{256}\right)$$