VSZ 1., 2018 NOV 22, ZH 2.

① $Z_1 \sim N(55, 4^2)$ $K \sim N(45, 3^2)$

a) $\mathbb{P}(K > Z_1) = \mathbb{P}(Z_1 - K < 0) =$ ☺

$(Z_1 - K) \sim N(10, 25)$

$\Phi(2)$

☺ $= \mathbb{P}\left(\frac{Z - K - 10}{5} < \frac{-10}{5}\right) = \Phi(-2) = 1 - 0.9772$

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b) $P_K := \mathbb{P}(K > 40) = \mathbb{P}(K - 40 > 0) =$

$= \mathbb{P}(K - 45 > -5) = \mathbb{P}\left(\frac{K-45}{3} > -\frac{5}{3}\right) = 1 - \Phi\left(-\frac{5}{3}\right)$

$= \Phi\left(\frac{5}{3}\right) = 0.9525$

$P_Z := \mathbb{P}(Z > 50) = \mathbb{P}(Z - 50 > 0) =$ 0.8944

$= \mathbb{P}(Z - 55 > -5) = \mathbb{P}\left(\frac{Z-55}{4} > -\frac{5}{4}\right) = \Phi\left(\frac{5}{4}\right)$

$X \sim BIN(100, P_K)$ $Y \sim BIN(100, P_Z)$

$\mathbb{P}(X > Y) = \mathbb{P}\left(\frac{X - 100 \cdot P_K}{\sqrt{100}} > \frac{Y - 100 \cdot P_Z}{\sqrt{100}} + \frac{100 \cdot (P_Z - P_K)}{\sqrt{100}}\right) =$ ☺

-0.581

$N(0, P_K \cdot (1 - P_K)) \sim X^*$ $Y^* \sim N(0, P_Z \cdot (1 - P_Z))$

0.0452 0.0944

DE MOIVRE-LAPLACE

1.OLDAL

$$\text{(:)} = P(\underbrace{X^* - Y^*}_{N(0,\sigma^2)} > -0.581) = \text{(☆)}$$

$$\sigma = \sqrt{0.0452 + 0.0944} = 0{,}374$$

$$\text{(☆)} = P\left(\frac{X^* - Y^*}{0.374} > \frac{-0.581}{0.374}\right) = 1 - \Phi(-1.55) =$$

$$= \Phi(1.55) = 0.9394$$

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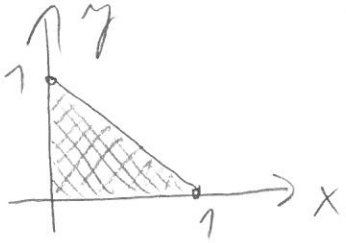
(2) TARTOMÁNY:



SZIMMETRIA

a) $$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = \int_0^1\int_0^{1-y} A\cdot(x+y)\,dx\,dy =$$

$$= 2\cdot A\cdot\int_0^1\int_0^{1-y} y\,dx\,dy = 2\cdot A\cdot\int_0^1 y\cdot(1-y)\,dy =$$

VEGE

$$= 2\cdot A\cdot\int_0^1 y - y^2\,dy = 2\cdot A\cdot\left(\frac{1}{2}-\frac{1}{3}\right) = \frac{A}{3} = 1$$

TEHÁT: $\boxed{A=3}$

b) SZIMMETRIA MIATT: f_X UGYANAZ, MINT f_Y

NA $0<x<1$: $f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^{1-x} 3\cdot(x+y)\,dy$

$$= 3\cdot\left[xy + \frac{y^2}{2}\right]_{y=0}^{y=1-x} = 3\cdot\left(x\cdot(1-x) + \frac{(1-x)^2}{2}\right)$$

2. OLDAL

$$f_X(x) = 3 \cdot \left(x \cdot (1-x) + \frac{(1-x)^2}{2} \right) \cdot \mathbb{I}[0 < x < 1]$$

(c) $$f_{Y|X}\left(y \mid \tfrac{1}{2}\right) = \frac{f\left(\frac{1}{2}, y\right)}{f_X\left(\frac{1}{2}\right)} = \frac{3 \cdot \left(\frac{1}{2} + y\right) \cdot \mathbb{I}\left[0 < y < \frac{1}{2}\right]}{3 \cdot \left(\frac{1}{2} \cdot \left(1 - \frac{1}{2}\right) + \frac{(1/2)^2}{2}\right)}$$

$$= \frac{8}{3} \cdot \left(\frac{1}{2} + y\right) \cdot \mathbb{I}\left[0 < y < \frac{1}{2}\right],$$ DE EZ IGAZIBÓL NEM IS KELLETT, MIVEL:

$$\mathbb{P}\left(X > Y \mid X = \tfrac{1}{2}\right) = \mathbb{P}\left(Y < \tfrac{1}{2} \mid X = \tfrac{1}{2}\right) = 1$$

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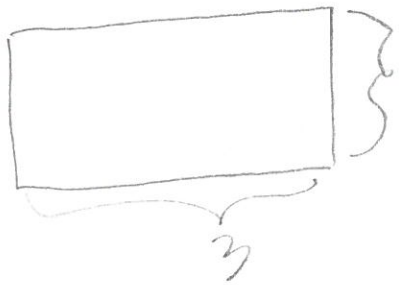
(3)



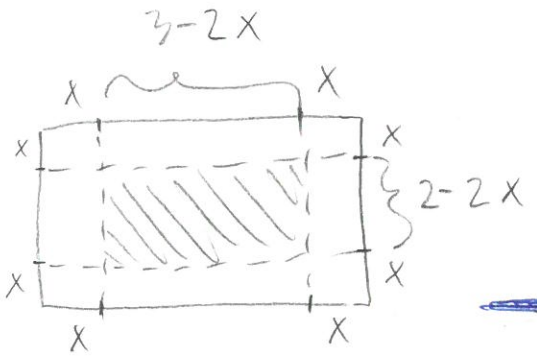
$$\mathbb{P}(0 \le \xi \le 1) = 1$$

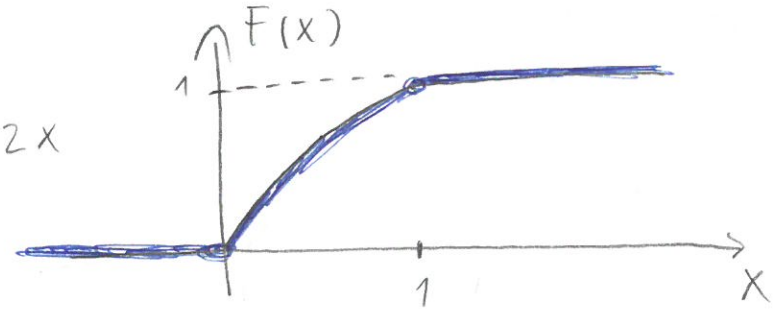
HA $x \in (0,1)$, AKKOR $F(x) = \mathbb{P}(\xi < x) =$

$$1 - \mathbb{P}(\xi \ge x) = 1 - \frac{(3-2x)\cdot(2-2x)}{6}$$

← KERÜLŐ TERÜLET

← ÖSSZES TERÜLET





$$\mathbb{E}(\xi) = \int_0^\infty \mathbb{P}(\xi \ge x)\,dx = \int_0^1 \frac{(3-2x)\cdot(2-2x)}{6}\,dx = \frac{7}{18}$$

3. OLDAL

(4) $X_{1,r} := \mathbb{1}[A_r]$, $A_r := \{r\text{-NÁL ILLESZKEDÉS}\}$

$Y_{1,r} := \mathbb{1}[B_r]$, $B_r := \{10+r\text{-NÁL }-\!\!\!''\!\!\!-\}$

$Z_{1,r} := \mathbb{1}[C_r]$, $C_r := \{20+r\text{-NÁL }-\!\!\!''\!\!\!-\}$

$$X_1 = \sum_{r=1}^{10} (X_{1,r} + Y_{1,r} + Z_{1,r})$$

$$\mathbb{E}(X_{1,r}) = \mathbb{E}(Y_{1,r}) = \mathbb{E}(Z_{1,r}) = \frac{3}{30} = \frac{1}{10}$$

$$\mathbb{P}(A_r) \quad \mathbb{P}(B_r) \quad \mathbb{P}(C_r)$$

$$\mathbb{E}(X_1) = 30 \cdot \frac{1}{10} = 3$$

$\mathrm{Var}(X_1) =$ 900-TAGÚ ÖSSZEG: KOVARIANCIÁI

$$\mathrm{Var}(X_{1,r}) = \mathrm{Var}(Y_{1,r}) = \mathrm{Var}(Z_{1,r}) = \frac{1}{10} \cdot \left(1 - \frac{1}{10}\right)$$

$$\mathrm{Cov}(X_{1,r}, X_{1,\ell}) = \frac{3}{30} \cdot \frac{3}{29} - \left(\frac{1}{10}\right)^2, \text{ HA } r \neq \ell$$

$$\mathrm{Cov}(X_{1,r}, Y_{1,r}) = \frac{3}{30} \cdot \frac{2}{29} - \left(\frac{1}{10}\right)^2$$

$$\mathrm{Cov}(X_{1,r}, Y_{1,\ell}) = \frac{3}{30} \cdot \frac{3}{29} - \left(\frac{1}{10}\right)^2, \text{ HA } r \neq \ell$$

$$\mathrm{Var}(X_1) = 30 \cdot \left(\frac{1}{10} \cdot \frac{9}{10}\right) + 30 \cdot 6 \cdot \left(\frac{3}{30} \cdot \frac{2}{29} - \left(\frac{1}{10}\right)^2\right) + \\ + (900 - 30 - 30 \cdot 6) \cdot \left(\frac{3}{30} \cdot \frac{3}{29} - \left(\frac{1}{10}\right)^2\right)$$

4. OLDAL