Optimization Algorithms for Large-Scale Unit Commitment Problems in Medium-Term Energy System Simulations

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The Project

Joint project with the Italian *Energy System Research Centre* (RSE s.p.a.)

Developing with RSE a simulator for *Energy Systems*.

Energy systems at a glance:

- producers share demand via a **global energy exchange**
- system divided in **zones** connected by a **capacitated network**
- system interacts with outer systems via **import/export** flows

Unit Commitment Problem

Planning thermoelectric energy production to satisfy energy demand at minimum cost

Problem Description

Decision variables:

- thermoelectric and hydroelectric productions
- energy exchanges between zones

Parameters:

- demand, zone by zone
- network capacity
- other non-dispatchable sources (wind, PV, waste...)
- import-export with outer systems

Objective:

- minimize total production cost
  - thermoelectric production costs (fuel, maintenance, pollution).
  - hypothesis: under a competitive market prices are minimized at clearing level

Thermoelectric Plants (TPP)

- discrete activation pattern (on-off state)
- linear cost function
  - non-linear formulation is too inefficient

Let x be the production level, y the state on/off (binary). Cost function is

$$f(x) = cx + ey$$

where c = marginal cost, e = fixed cost

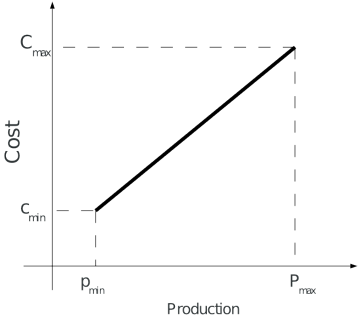

Figure : Cost function for thermoelectric plants

Model entities

Thermoelectric Plants (TPP):

- linear cost function
- discrete activation pattern

Hydroelectric Plants (HPP):

- costless production
- linear continuous behaviour

Network:

- continuous network flow

Time:

- simulation over 1 year with hourly resolution (= 8760 1h-periods)

$\Rightarrow$ A large-scale MILP model.

Literature

In literature:

- short-term models $\rightarrow$ operational decisions
  - non-linear formulations (Frangioni et al '03, '06, '13)
- medium-term models $\rightarrow$ tactical decisions
  - mixed integer linear formulations and heuristics (Chang et al '04)
- long-term models $\rightarrow$ strategic decisions
  - mixed integer linear model
  - In Kjeldsen and Chiarandini ('12) annual simulation of Danish market. Solved via constructive heuristics.

Modelling

Variables:

- thermoelectric plants $\rightarrow$ state: binary, production: continuous
- hydroelectric plants $\rightarrow$ production: continuous
- network $\rightarrow$ flows: continuous

Constraints:

- upper and lower bounds on plants production
- upper and lower bounds on hydro plants reservoir levels
- hydroelectric energy balance: for each hydro plant and period water in = production + reservoir increase + spillage
- zonal energy balance: for each period and zone

$$\text{demand} + \text{export flow} = \text{energy production} + \text{import flow}$$

- minimum up/down constraints for thermal plants
  After switching state the thermal plant must mantain the new state for τ periods.

Section 2

Preprocessing

Grouping TPPs

In each zone TPPs divided in **groups** of units with the same *marginal cost* c.
In each group TPPs divided in **subgroups** of units with the same *fixed cost* e, same *technical minima and maxima* and *minimum up/down constraints*.

For each period $t \in T$, zone $z \in Z$, group $g \in G_z$ the group cost function is:

$$c_{tzg} x_{tzg} + \sum_{m \in M_{zg}} e_{tzgm} y_{tzgm}$$

with parameters

- c_{tzg} marginal cost
- e_{tzgm} fixed cost

and variables

- $x_{tzg} \geq 0$ total group production
- $y_{tzgm} \in \{0..k_m\}$ number of active plants in subgroup $m \in M_{zg}$ (integer)

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A discontinuous piece-wise linear function.

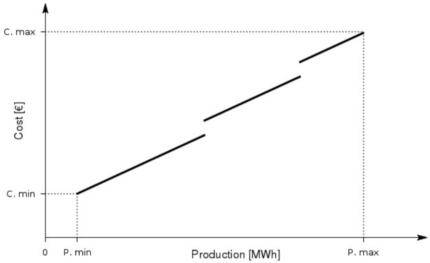

Figure: Lower envelope of the group cost function

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$\Rightarrow$ **Result:** less symmetry, more efficiency!

Section 3

Heuristics

Commit&Dispatch Algorithm (C&D)

0. Estimate: estimate *zonal production levels* of TPP X^i_{tz} for each period and zone
1. Commit: **for each zone** $z \in Z$ compute commitment $(y^i_{tzgm})_z$ to satisfy production levels X^i_{tz}
2. Dispatch: compute optimal dispatching according to commitment (y^i_{tzgm})
   Get a feasible solution with commitment (y^i_{tzgm}) and production levels (x^i_{tzg})
3. Repeat: If the new solution is different from the previous one let $X^{i+1}_{tz} = \sum_{g \in G_z} x^i_{tzg}$ $\forall t \in T, z \in Z$, $i := i+1$, and go to **Commit**. Otherwise STOP.

Key points:

- after each iteration a feasible solution
- at each iteration the new solution is not worse than the old one (non-strict monotonicity)
- finitely converges to a solution, not necessarily the optimum

Estimate

At first the algorithm estimates zonal production for TPPs.

Feasibility: for every period and zone, the estimated zonal production must

- be between 0 and the maximum total production level of the group
- satisfy the zonal energy balance constraint
  demand + export flow = energy production + import flow

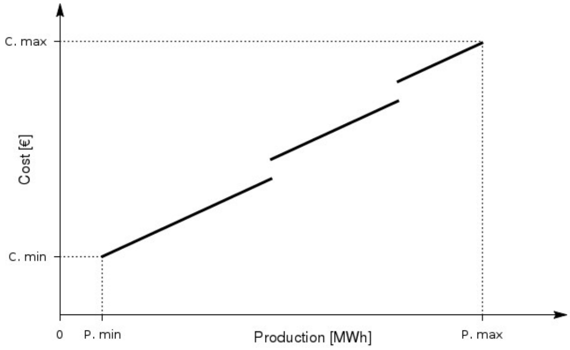
To estimate zonal production at step 0th we consider two **continuous lower bounds** for UCP:

- the *Continuous Relaxation* (CR)
- the *Aggregated Continuous Relaxation* (ACR).

ACR

Aggregated Continuous Relaxation (ACR)

For each group we construct the best line (passing by the origin) which underestimates the original group cost function (requires linear complexity)



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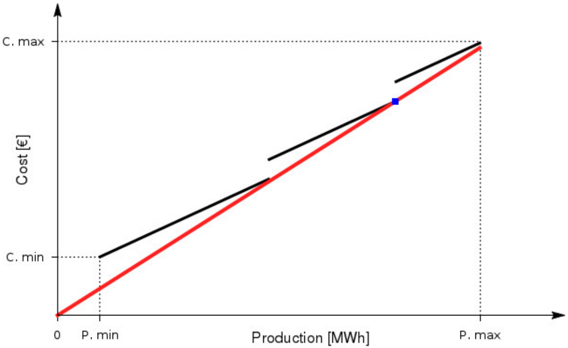

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Compared to the continuous relaxation (CR) of UCP the ACR

- is **smaller**, y variables are removed.
  In the Italian case from 148 TPPs we get 98 groups and 103 subgruops.
- can be formulated as a Network Flow Problem and solved in polynomial time
- it is provably weaker

Section 4

Exact Approaches

Column Generation (CG)

- For each subgroup $m \in M_{zg}$ consider every feasible commitment $u \in S_{zgm}$ (**pattern**)
- define α_u binary variable to select $(\alpha_u = 1)$ pattern u.
- Master Problem: new formulation of UCP with patterns. Substitute state variables (y_{tzgm}) with pattern variables $(\alpha_u)_{zgm}$.

Constraints on patterns (*)

$$\sum_{\substack{m \in M_{zg}, \\ u \in S_{zgm}}} \hat{p}_{tzgmu} \alpha_u \leq x_{tzg} \leq \sum_{\substack{m \in M_{zg}, \\ u \in S_{zgm}}} \hat{P}_{tzgmu} \alpha_u \quad \forall t \in T, z \in Z, g \in G_z$$

$$\sum_{u \in S_{zgm}} \alpha_u = 1 \qquad \forall z \in Z, g \in G_z, m \in M_{zg}$$

We actually consider a master with a limited selection of patterns (**Restricted Master Problem (RMP)**)

Column Generation Algorithm

Scheme:

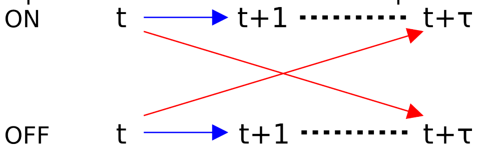
0. Init: initialize RMP with patterns from a feasible solution
1. Relax: solve continuous relaxation of RMP (lower bound)
2. Rounding: choose for each subgroup the pattern with highest α_u, obtaining a feasible solution (upper bound).
3. Pricing: for each subgroup generate the pattern of minimum reduced cost according to dual prices of (*) in current RMP solution.
4. If new patterns are generated go to **Relax**, otherwise go to **Enumerate**
5. Enumerate: if the gap between the best lower and upper bounds is positive and *small enough*
   - a enumerate all the columns whose reduced cost is smaller than the gap (via Constraint Programming)
   - b solve RMP as integer program

   Otherwise if the gap is positive perform Branch&Bound, else STOP.

Pricing

An ILP for each subgroup in the y variables.
Solvable in linear-time with known dynamic programming scheme (see Frangioni et al '06 for reference).

Possible commitments as a graph.
Optimal commitment is a shortest path on the graph.



blue: keep the current state and go to $t+1$
red: switch to a new state for at least τ periods

The algorithm can manage only one plant at time. In our case we have more plants in each subgroup.

However we can show that at each period it is optimal to switch either all or none of the plants of the subgroup.
Pricing objective function:

$$\phi_{\mathrm{Pricing}}(y) = \sum_{t \in T} (e_t - \lambda_t - \mu_t) y_t - \eta$$

where λ_t, μ_t and η are the dual prices of constraints (*)

Partial Branch&Bound (PB&B)

- Idea: remove most significant violations in the *continuous relaxation* to reduce *the gap* with *optimal solution*
- Hypothesis: most significant violations are determined by **under-min production**

Under-min production

Production *strictly* between 0 and the group minimum

$$0 < x_{tzg} < \min_{m \in M_{zg}} \{p_m\}$$

Partial Branch&Bound: Algorithm

Consider continuous relaxation model CR

1. Solve: solve the model.
   Let $R = \{t \in T, z \in Z, g \in G_z : 0 < x_{tzg} < \min_{m \in M_{zg}} \{p_m\}\}$.
2. If $R = \emptyset$ $\textsc{Stop}$. Otherwise go to **Select**
3. Select: compute mean *under-min* production ρ

$$\rho = \frac{\sum_{t,z,g \in R} x_{tzg}}{|R|}$$

   Let $C = \{t \in T, m \in M_{zg} : \rho \leq x_{tzg} < \min_{m_1 \in M_{zg}} \{p_{m_1}\}\}$
4. Cut: restore integrality constraints for $y_{tzgm} : (t, m) \in C$ in the model. Go to **Solve**

At every iteration new integrality constraints are restored, strengthening the relaxation.

Section 5

Results

Test

Instance: Italy 2011.

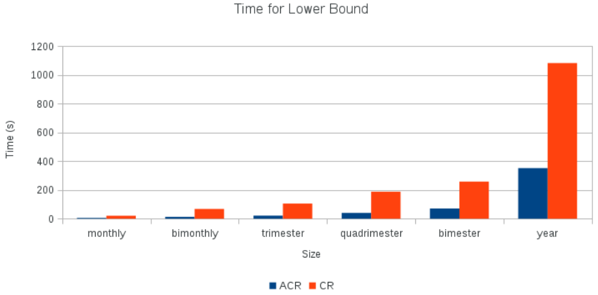
For simplicity we **relax minimum up/down** constraints (see later).

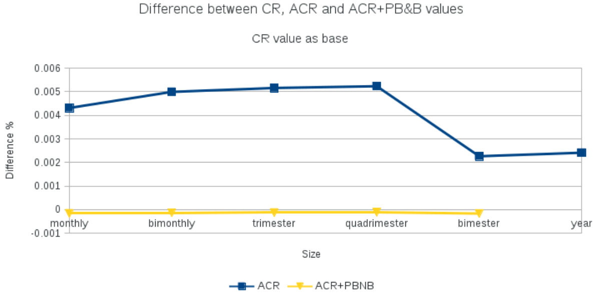
Scheme:

1. Commit&Dispatch $\times$ 1 iteration, initialized with either ACR, CR or ACR+PB&B
   - time limit for PB&B: 1h
2. Column Generation initialized with heuristic C&D solution.
   Pricing solved as MIP.
   - time limit for CG: 1h

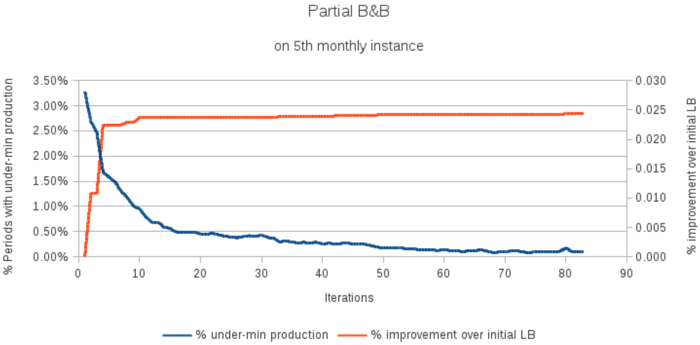
Implementation: CPLEX 11 + AMPL on dual core with 4G RAM.

Lower Bounds

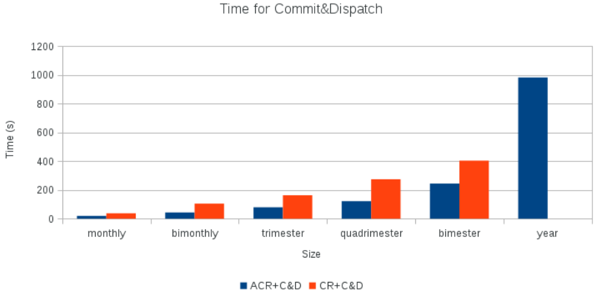


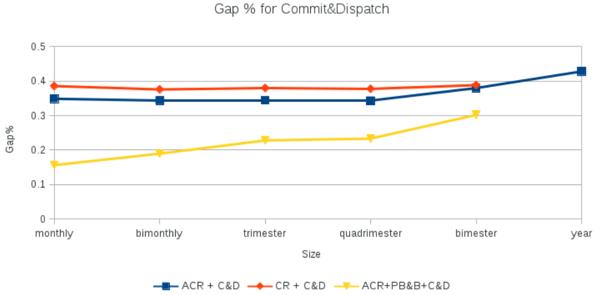


Partial B&B

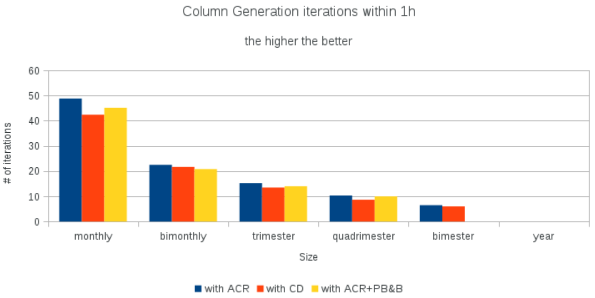

Partial B&B
on 5th monthly instance
% Periods with under-min production
3.50%
3.00%
2.50%
2.00%
1.50%
1.00%
0.50%
0.00%
0.030
0.025
0.020
0.015
0.010
0.005
0.000
% improvement over initial LB
0
10
20
30
40
50
60
70
80
90
Iterations
% under-min production
% improvement over initial LB

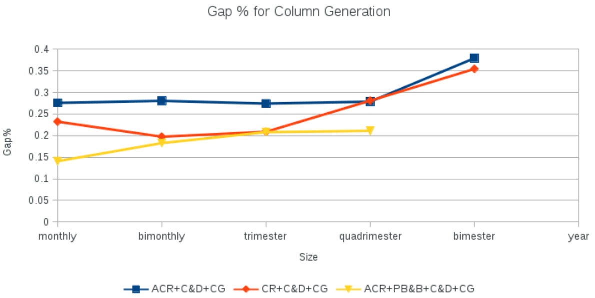
Commit&Dispatch





Column Generation





Instances sizes

Size	Id	Size after presolving Constraints	 Continuous var.	 Integral var.	 Binary var.
1 month	1	152,541	110,184	13,870	67,158
	2	152,360	110,050	13,853	67,104
	3	151,649	109,485	13,750	66,852
	4	151,578	109,451	13,740	66,836
	5	151,624	109,477	13,753	66,831
	6	152,406	110,075	13,864	67,112
	7	152,266	109,956	13,855	67,062
	8	152,492	110,149	13,861	67,150
	9	152,147	109,879	13,819	67,013
	10	152,218	109,932	13,829	67,043
	11	152,311	109,992	13,838	67,093
	12	151,774	109,591	13,813	66,908
2 months	1	304,887	220,249	27,721	134,246
	2	302,960	218,771	27,478	133,544
	3	304,033	219,579	27,616	133,931
	4	304,776	220,149	27,725	134,199
	5	304,423	219,867	27,646	134,080
	6	303,740	219,332	27,595	133,864
3 months	1	456,617	329,831	41,493	201,113
	2	455,838	329,210	41,386	200,855
	3	457,109	330,187	41,563	201,281
	4	456,073	329,383	41,460	200,918
6 months	1	912,782	659,305	82,904	402,076
	2	913,597	659,919	83,019	402,375
12 months	1	1,826,563	1,319,378	165,951	804,489

Results - Table

Size	Id	Continuous Lower Bounds ACR Time (s)	CR Time (s)	C&D with ACR Time (s)	Gap %	with CR Time (s)	Gap %	with ACR+PB&B Gap %	C&D+CG with ACR Gap %	with CR Gap %	with ACR+PB&B Gap
1 month	1	5	23	19	0.44	38	0.47	0.22	0.27	0.21	0.16
	2	6	19	18	0.34	36	0.38	0.14	0.34	0.24	0.14
	3	5	19	19	0.31	37	0.33	0.13	0.31	0.15	0.13
	4	6	15	20	0.41	32	0.42	0.15	0.41	0.41	0.15
	5	5	17	17	0.34	33	0.36	0.14	0.34	0.19	0.14
	6	8	19	21	0.35	36	0.36	0.18	0.23	0.12	0.18
	7	7	21	20	0.30	39	0.36	0.12	0.15	0.14	0.09
	8	6	29	23	0.39	47	0.47	0.21	0.22	0.47	0.21
	9	6	25	19	0.32	41	0.35	0.16	0.32	0.11	0.16
	10	5	24	19	0.34	40	0.40	0.14	0.34	0.33	0.14
	11	5	20	20	0.32	37	0.37	0.15	0.21	0.21	0.15
	12	5	19	18	0.32	36	0.35	0.14	0.18	0.21	0.04
2 months	1	13	62	42	0.41	99	0.43	0.20	0.28	0.21	0.16
	2	13	60	41	0.33	96	0.37	0.16	0.31	0.14	0.16
	3	11	70	41	0.33	108	0.36	0.17	0.33	0.22	0.17
	4	15	59	58	0.36	97	0.41	0.24	0.29	0.18	0.24
	5	13	76	39	0.31	112	0.34	0.17	0.20	0.27	0.17
	6	12	80	39	0.33	117	0.35	0.18	0.28	0.17	0.18
3 months	1	20	130	68	0.37	188	0.40	0.30	0.25	0.19	0.27
	2	17	114	78	0.34	171	0.40	0.19	0.34	0.26	0.14
	3	29	82	96	0.36	142	0.38	0.23	0.27	0.26	0.23
	4	20	95	75	0.30	152	0.35	0.19	0.23	0.13	0.19
4 months	1	33	162	98	0.39	251	0.40	0.27	0.28	0.26	0.24
	2	55	199	152	0.31	285	0.37	0.23	0.31	0.28	0.21
	3	33	202	115	0.33	286	0.37	0.20	0.24	0.30	0.17
6 months	1	55	292	216	0.39	434	0.41	0.27	0.39	0.41	-
	2	87	224	273	0.37	374	0.37	0.34	0.37	0.30	-
1 year	1	352	1083	983	0.43	-	-	-	-	-	-

Notes

- ACR provides a good lower bound with high efficiency
- integer problems **Commit** and **Pricing** separable in small, easy subproblems
  - solved with no branch&bound, sometimes a few MIP simplex iterations
- CG and PB&B reduce gaps mostly at first iterations.

Notes (cont.)

Compared with Kjeldsen and Chiarandini (K&C) ('12)

- we solve a much larger model with comparable time and precision
  - K&C best configuration solves yearly instance with 20 TPPs within 15 min and 1% gap
  - Commit&Dispatch solves yearly instance with 148 TPPs within 20 min and 0.5%gap
  - Our model includes HPPs and network
  - In both cases minimum up/down constraints are not handled
- our algorithm does not have parameters to tune

However we do not model ignition costs.

Conclusions

To compute more realistic and meaningful solution we need to test the algorithms with **minimum up/down constraints** for TPPs.
With these constraints

- the model is much more expensive to solve
- ACR and CR provide looser bounds

Example on quadrimester instances

- without up/down cons.: 0.35% gap within 3 min with ACR+C&D, 0.30% gap with 1h CG
- with up/down cons.: 3% gap within 20 min with ACR+C&D, 2% gap with 1h CG

On the other hand the decomposition schemes we devised are general and can already be solved efficiently with high accuracy.