
Laureano F. Escudero
Universidad Rey Juan Carlos, Móstoles (Madrid), Spain

Workshop on Mathematical Models and Methods for Energy Optimization
EU COST-Action TD1207
25-26 September 2014, Budapest
Table of Contents

- Aim and scope: Energy Generation Capacity and Transmission Expansion Planning EGCTEP
- Uncertainty in EGCTEP
- Multicriteria in EGCTEP
- Math optim under uncertainty. Risk neutral multistage stochastic mixed 0-1 modeling
- EGCTEP approximating modeling (since the problem is SMINP): A gigantic but well structured multicriteria multistage stochastic mixed 0-1 linear optimization problem with risk management. Elements:
  - Dynamic setting
  - Site location and capacity decisions
  - Replicated networks (hydro valleys)
  - General networks: (current and candidate power generation plants, energy transmission nodes, demand nodes)
  - Stochastic optimization (deep inside on the scenario tree).
- Risk averse extension: Time Stochastic Dominance (TSD).
- Brief. ref. to Decomposition methods for multistage stochastic optimization problems.
Aim and scope

Presenting a model for addressing challenges for a long term (e.g., 30 years time horizon) power generation capacity and energy transmission network infrastructure expansion planning along the years of the time horizon at pan-European level.

Scope: Helping to decide on:

1. Type and mix of power generation sources (ranging from less coal, nuclear and combined cycle gas turbine to more renewable sources: hydroelectric, wind, solar, photovoltaic and biomass)
2. New power generation plant / farm location and capacity
3. Location and capacity of new lines in the transmission network
Goals to achieve. Helping on quantifying

- Satisfying electricity demand from main focal points in the European region
- Eliminating existing technological and political barriers
- Maximizing different types of utility criteria at pan-European level
- Benefits of using cleaner, safer and efficient (cheaper) energy accessible to all the consumption nodes from perhaps far away power generation sites in the network.
Wind energy could be generated from the North Sea and South of Spain,

Solar energy can be generated from South Spain and South Portugal,

Biomass from neighbors to the European region and

all together can be used for satisfying electricity at multiple European points far away from the physical sources.
Uncertainty in EGCTEP

Main parameters:
- Market electricity demand and prices at the network nodes of the energy system.
- Operating hours per period of power generation technologies.
- $CO_2$ emission permits and Green Certificates prices and allowed bounds.
- Power generation cost of different technologies.
- Electricity loss of new transmission technologies.
- Characteristics (i.e., maximum energy flow and reactance) of cable types on new energy transmission lines.
- Fixed and variables costs of total power generation and energy transmission technologies.
Representation: Multistage non-symmetric scenario tree (whose stages are included by consecutive years whose constraint systems must be satisfied in an individual basis).

Gigantic scenario tree: E.g., Brazilian power systems: 120 periods, \(20^{119}\) scenarios (Sagastizabal MP’12).

A combination of Sample Scenario Schemes, inexact scenario groups Decomposition algorithms and High Performance Computing is a ’must’.
Multicriteria in EGCTEP

Maximizing NPV of expected investment and consumer stakeholders goals over the scenarios along the time horizon subject to risk reduction of the negative impact of non-wanted scenarios on multiple types of utility objectives and stakeholders at European level:

- Maximizing power share of cleaner, safer and efficient -cheaper- energy accessible to all consumption nodes.
- Generation and Transmission Network reliability.
- EC directives on environmental issues and others.
- EU governments, etc.
Some facts on European Renewable Energy Sources (RES) Generation and Transmission systems

- EU has established aggressive emission reduction targets: a 20% reduction in greenhouse gases with respect to 1990 levels by 2020 (most of the member countries are still far away from that target) and endorsing an objective of 80% reductions by 2050.

  See epp.eurostat.ec.europa.eu/portal/page/portal/eurostat/home/.

- So, vast amounts of new generation plants / farms are expected to be built in the medium term future. A substantial part of this new RES generation will probably materialize in the near future.

- Transmission investments are very capital intensive (e.g., ENTSO-e members joint budget EURO 104 bn, 2012-2022) and long useful life (up to 40 years).

  See www.entsoe.eu.
Some European project on RES Generation and Transmission systems

- Desertec, project of a German consortium for installing RES power plants (over 20 GW, photovoltaic, solar power, wind, ...) in Sahara desert to be connected to European transmission system. See www.Desertec.org/fileadmin/downloads/desertec
- OWF: 5.3 GW installed in Europe (including London array, 1 GW). See EWEA. Wind in power. report 2012.
- NSCOGI. See www.The North Sea Countries’ Offshore Grid Initiative.
Deterministic mixed 0-1 optimization model

\[ z_{EV} = \max \sum_{t \in \mathcal{T}} (a^t x^t + b^t y^t) \]

s.t. \[ \sum_{t' \in \mathcal{A}^t} (A_i^{t'} x^{t'} + B_i^{t'} y^{t'}) = h^t \quad \forall t \in \mathcal{T} \]  
\[ x^t \in \{0, 1\}^{n_x(t)}, \ y^t \in \mathbb{R}^{n_y(t)} \quad \forall t \in \mathcal{T}. \]
A **stage** of a given horizon is a set of consecutive time periods where the realization of the uncertain parameters takes place.

A **scenario** is a realization of the uncertain parameters along the stages of a given horizon.

**Scenario group** for a given stage is the set of scenarios with the same realization of the uncertain parameters up to the stage.

**Multistage scenario tree.**
Ω = Ω^1 = \{10, 11, \ldots, 17\}; Ω^2 = \{10, 11, 12\}

G = \{1, \ldots, 17\}; G_2 = \{2, 3, 4\}

A^{17} = \{1, 4, 9, 17\}

Figura: Multistage nonsymmetric scenario tree
**Scenario tree notation**

\[ \mathcal{E}, \text{ set of the stages along the horizon.} \]

\[ \Omega, \text{ set of scenarios.} \]

\[ \mathcal{G}, \text{ set of scenario groups.} \]

\[ \Omega^g \subseteq \Omega, \text{ set of scenarios in group } g, \text{ for } g \in \mathcal{G}. \]

\[ \mathcal{G}^e \subseteq \mathcal{G}, \text{ set of scenario groups for stage } e, \text{ for } t \in \mathcal{T}. \]

\[ e(g), \text{ stage to which scenario group } g \text{ belongs to, for } g \in \mathcal{G}. \]

\[ w^\omega, \text{ likelihood or weight assigned by the user to scenario } \omega \in \Omega. \]

\[ w^g, \text{ weight assigned by modeler to scenario group } g \in \mathcal{G}. \text{ It is computed as } w^g = \sum_{\omega \in \Omega} w^\omega \]
Scenario tree notation (c.)

\( \mathcal{T} \), set of periods (usually, years) in the time horizon, where last period \( T = |T| \).

\( \mathcal{T}^e \), set of (consecutive) periods in stage \( e \), for \( e \in \mathcal{E} \), \( \mathcal{T}^e \cap \mathcal{T}^{e'} = \emptyset \), \( e, e' \in \mathcal{E} : e \neq e' \), \( \mathcal{T} = \bigcup_{e \in \mathcal{E}} \mathcal{T}^e \).

\( t(e) \), first period in set \( \mathcal{T}^e \), for \( e \in \mathcal{E} \).

\( \mathcal{A}^g \), set of ancestor nodes (scenario groups) in the scenario tree to node (scenario group) \( g \) (including itself), for \( g \in \mathcal{G} \), so, there is underlined a directed graph where \( |\mathcal{G}| \) is the set of nodes.

\( \mathcal{S}^g \), set of successor nodes (scenario groups) in the scenario tree to node (scenario group) \( g \), for \( g \in \mathcal{G} \).

Note: The set of scenarios \( \Omega^g \) of any group \( g \) from last stage is a singleton. Let us assume that \( \omega = g \) for \( \omega \in \Omega^g \) and \( g \in \mathcal{G}^{|\mathcal{E}|} \).

Let \((t, g)\) denote the pair of indexes for period \( t \) and scenario group \( g \), for \( t \in \mathcal{T}^e(g) \), \( g \in \mathcal{G} \).
Example of four-stage scenario tree (15 nodes, 8 scenarios)
- $a^{t,g}_1$, $b^{t,g}_1$, vectors of objective function coeffs. for variables in vectors $x^{t,g}, y^{t,g}$, res.
- $h^{t,g}$, rhs for set of constraints related to group $g$ in period $t$, for $t \in T^{e(g)}$, $g \in G$.
- $A^{t',g'}_{t,g}$, $B^{t',g'}_{t,g}$, matrices related to ancestor pair $(t', g')$ in constraints related to pair $(t, g)$.
\[ z_{RN} = \max \sum_{g \in G} \sum_{t \in T^e(g)} w^g (a^t,g x^t,g + b^t,g y^t,g) \]

subject to

\[
\sum_{g' \in A^g} \sum_{t' \in T^e(g')} (A_{t,g}^{t',g'} x^{t',g'} + B_{t,g}^{t',g'} y^{t',g'}) = h^{t,g} \quad \forall t \in T^e(g), \ g \in G
\]

\[ x^{t,g} \in \{0, 1\}^{nx(t,g)}, \ y^{t,g} \in \mathbb{R}^{ny(t,g)} \quad \forall t \in T^e(g), \ g \in G \]

Note: The constraint qualification \( t' \leq t \) is only active for \( g' = g \).
Approximation: A huge but very well structured multicriteria multistage stochastic mixed 0-1 linear optimization problem
Key energy stakeholders (producers, transmission system operators (TSO), distributors) want to determine their optimal planning for investment in power generation capacity and energy transmission in a long term horizon.

The power producers and TSOs may have different objectives in a liberalized electricity market, where rules are issued by.

Regulatory Authorities aim: Promoting the development of RES (i.e. by geothermal, wind, biomass and hydro power plants) for power generation systems with reduced $CO_2$ emissions and penalization.
Green Certificate schemes support power generation from RES, and penalizes generation from conventional power plants (i.e. CCGT, coal and nuclear power plants).

Every year a prescribed ratio is required between the electricity produced from RES and the total produced one.

In case the actual ratio attained at a given year does not exceed the prescribed one, the power producer has to buy Green Certificates in order to satisfy the related constraint.

On the contrary, when the actual ratio attained is greater than the prescribed one, the power producer can sell Green Certificates in the market.
Power producers’ aims: Maximizing expected profit along the time horizon at NPV subject to feasible constraints for nodes (i.e., scenario groups) in the scenario tree under a risk averse time stochastic dominance (TSD) strategy.

Revenues from sale of electricity depend on the market price and the amount of electricity sold, which is bounded above by the power producer’s market share estimation in a competitive market. The revenues also depend on the number of operating hours per year of the power plants / farms in the generation system.

Penalization for emitting $CO_2$ greatly varies among generation technologies.

Variable and fixed costs also greatly differ among the generation and line transmission technologies.

Investment costs on some types of technologies depend on the plant rated power and on the investment costs per power unit.
Revenues and costs associated to the Green Certificate scheme depend on the Green Certificate price, as well as on the yearly ratio between generation from RES and total annual generation.

The evolution of electricity prices in nodes covered by the energy network along the time horizon is not known at the time when the investment decisions are to be made. Therefore a risk is associated with the expected profit from power generation capacity and energy transmission expansion, due to the uncertainty on the main parameters.
The proposed model determines the evolution of the power generation mix and location and energy transmission network along the time horizon. So, it determines for every power generation technology and every transmission line technology:

- site location of each power plant and transmission line
- year to start the construction

depending on the scenario group along the time horizon.

It could be possible that at the year when the new power plant or the new energy transmission line is ready for being in operation, the realization of uncertain prices and electricity demand and other key uncertain elements be drastically changed along the scenario tree from the scenario group where the construction have started.
EGCTEP SMILP approximating model due to its SMINLP complexity and long time horizon

- Linear functions approximate nonlinearities on:
  - Investment and operation costs for generation and transmission
  - Hydropower generation
  - Windpower generation
  - Power flow
  - Generation and flow losses, etc.

- Unit commitment (scheduling problem) has been oversimplified in the model.

- Some peculiarities of the transmission system are relaxed. They are related to substations, cables, transformers and converters types and their features, peculiarities of Off-shore Wind Farms, sets of AC and DC cables, correspondence between transformer or converter types and their respective voltage levels, etc.
$\mathcal{N}^T$, Candidate technologies for thermal power generation.

$\mathcal{N}^R$, Candidate technologies for power generation from RES (without considering power generation from hydro valleys since it has a different treatment).

$\mathcal{N}$, Candidate technologies (i.e., $\mathcal{N} = \mathcal{N}^T \cup \mathcal{N}^R$).
EGCTEP problem modeling. Power generation sets (c.)

\[ \mathcal{I}^{n^T}, \] Set of potential thermal power plants of technology \( n \), for \( n \in \mathcal{N}^T \) that can be constructed.

\[ \mathcal{I}^{N^T} = \bigcup_{n \in \mathcal{N}^T} \mathcal{I}^{n^T}, \] Set of potential thermal power plants.

\[ \mathcal{I}^{n^R}, \] Set of potential power RES plants of technology \( n \), for \( n \in \mathcal{N}^R \) that can be constructed.

\[ \mathcal{I}^{N^R} = \bigcup_{n \in \mathcal{N}^R} \mathcal{I}^{n^R}, \] Set of potential RES power plants (without considering hydro power generation).

\[ \mathcal{I}^{N} = \mathcal{I}^{N^T} \cup \mathcal{I}^{N^R}, \] Set potential power plants.

Note: Each new plant will be sited in a given already chosen location (i.e., node), if it is selected.

\[ n(i), \] Technology type of potential plant \( i \in \mathcal{N} \).
$\mathcal{I}^K$, Current thermal power plants (i.e., at period 0 of the time horizon).

$\mathcal{I}^{K^R}$, Current RES power plants (i.e., at period 0).

$\mathcal{I}^K = \mathcal{I}^{K^T} \cup \mathcal{I}^{K^R}$, Current power plants.

Note: It is assumed that the sites are unique for the currently owned and potential new plants, so, $\mathcal{I}^N \cap \mathcal{I}^K = \emptyset$. 
EGCTEP problem modeling. Hydropower generation sets

\[ \mathcal{V}, \text{ Hydropower valleys (with hyper period water stored).} \]

\[ \mathcal{I}^v, \text{ Reservoirs in hydropower valley } v \in \mathcal{V}. \]

\[ \mathcal{U}_i \subseteq \mathcal{I}^v, \text{ Upstream reservoirs to reservoir } i \in \mathcal{I}^v, v \in \mathcal{V}. \]

\[ \mathcal{D}_i \subseteq \mathcal{I}^v, \text{ Downstream reservoirs to reservoir } i \in \mathcal{I}^v, v \in \mathcal{V}. \]

Note: \[ \mathcal{I}^v = \{i\} \cup \mathcal{U}_i \cup \mathcal{D}_i. \]

\[ \tilde{\mathcal{D}}_i \subseteq \mathcal{D}_i, \text{ Reservoir } j, \text{ such that } \text{'canal'} \ ij \text{ has a potential increase of power generation over the current one (that even can be zero), for } i \in \mathcal{I}^v, v \in \mathcal{V}. \]

\[ \mathcal{I}^V = \bigcup_{v \in \mathcal{V}} \mathcal{I}^v, \text{ Reservoirs in the hydropower valleys.} \]
For each 'producer' genco \( \in \) GENCOS, where GENCOS is the set of 'producers' or potential 'producers' in the generation system:

\[
(I^{NT})_{genco} \subseteq I^{NT}, \quad \text{Set of potential new power plants whose 'rights' are owned by the 'producer'.}
\]

\[
(I^{NR})_{genco} \subseteq I^{NR}, \quad \text{Set of potential new RES power plants whose 'rights' are owned by the 'producer'.}
\]

\[
(I^{KT})_{genco} \subseteq I^{KT}, \quad \text{Set of actual thermal power plants owned by the 'producer'.}
\]

\[
(I^{KR})_{genco} \subseteq I^{KR}, \quad \text{Set of actual RES power plants owned by the 'producer'.}
\]

\[
(I^{V})_{genco} \subseteq \mathcal{V}, \quad \text{Set of hydropower valleys owned by the 'producer'.}
\]

\[
(I^{N})_{genco} = (I^{NT})_{genco} \cup (I^{NR})_{genco}.
\]

\[
(I)_{genco} = (I^{V})_{genco} \cup (I^{KR})_{genco} \cup (I^{KT})_{genco} \cup (I^{N})_{genco}.
\]
EGCTEP problem modeling. ET network sets

\( \mathcal{I}^{Tr} \), Pure energy transmission nodes (i.e., they are not power generation plants).

\( \mathcal{I} \), Nodes in the energy (connected) network, such that \( \mathcal{I} = \mathcal{I}^V \cap \mathcal{I}^K \cup \mathcal{I}^N \cap \mathcal{I}^{Tr} \).

For simplicity it is assumed that any node in the energy system can have energy demand.

\( \mathcal{L}^N \), Candidate (new) transmission lines to be installed in the energy network.

\( \mathcal{L}^K \), Current lines in the energy network.

\( \mathcal{L} \), Transmission lines in the energy network (i.e., \( \mathcal{L} = \mathcal{L}^K \cup \mathcal{L}^N \)).

\( C^{ij} \), Cables in transmission line \( ij \in \mathcal{L} \).
For each transmission system operator \( tso \) in \( TSOS \), where \( TSOS \) is the set of transmission system operators in the energy network:

\[
\mathcal{L}_{tso} \subseteq \mathcal{L},
\]

Transmission lines in the energy network operated or to be operated by \( tso \).
\( S_n \) \([t]\), Construction time periods required for any power plant of candidate technology \( n \in \mathcal{N} \) to be in generation.

\( L_i \) \([t]\), Industrial life of any power plant of candidate technology \( i \in \mathcal{N} \) or current power plant \( i \in \mathcal{I}^K \).

\( S_{ij}, L_{ij} \) \([t]\), Construction time periods required for hydro power generation turbine(s) in 'canal' \( ij \) and related industrial life, res., for \( j \in \mathcal{D}_i \), \( i \in \mathcal{I}^V \).

\( S_{ijc}, L_{ijc} \) \([t]\), Construction time periods required for cable \( c \in \mathcal{C}^{ij} \) of transmission line \( ij \) and related industrial life, res. for \( ij \in \mathcal{L}^N \) in the energy network.
EGCTEP problem modeling. Deterministic parameters (c.)

\( \tilde{T}^{n} \) [–], Maximum number of plants of candidate technology \( n \in \mathcal{N} \) that can be constructed along the time horizon.

\( P_{i} \) [MW], Rated power of any plant of candidate technology \( i \in \mathcal{N} \) or current power plant \( i \in \mathcal{I}^{K} \).

\( P_{i,t} \) [GWh], Minimum generated energy imposed for any plant of candidate technology \( i \in \mathcal{N} \) or current power plant \( i \in \mathcal{I}^{K} \) at period \( t \in \mathcal{T} \).
EGCTEP problem modeling. Deterministic parameters (c.)

\( \nu_i \) \([-]\), Percentage of loss of any power plant of candidate technology \( i \in \mathcal{N} \) or current power plant \( i \in \mathcal{I}^K \).

\( \zeta_i \) \([TM/GWh]\), \( CO_2 \) emission rate of any thermal power plant of candidate technology \( i \in \mathcal{N}^T \) or current power plant \( i \in \mathcal{I}^{K^T} \).

\( \lambda_{ijc} \) \([oh]\), Reactance of cable \( c \in \mathcal{C}^{ij} \) of current transmission line \( ij \in \mathcal{L}^K \) in the energy network.
$\overline{r}^K_{ij}$ [\(h^3\)], Water capacity for hydro power generation in current turbine(s) in 'canal' \(ij\), for \(j \in \mathcal{D}_i, i \in \mathcal{I}^V\).

$\phi^K_{ij}$ [\(MWh/h^3\)], Assumed constant converting one \(h^3\) into energy generated by the current (identical turbine(s)) in 'canal' \(ij\), for \(j \in \mathcal{D}_i, i \in \mathcal{I}^V\).
EGCTEP problem modeling. Deterministic economic parameters (c.)

\[ \rho \text{ [–], Discount rate.} \]

\[ M_i, R_i \text{ [MEURO/MW], Investment cost and its periodic allocation (see below), res., required by new power plant } i \in \mathcal{I}^{NT} \cap \mathcal{I}^{NR}. \]

\[ M_{ij}, R_{ij} \text{ [MEURO/MW], Investment cost and its periodic allocation (see below), res., required by new turbine(s) for hydro generation replacing the old ones in 'canal } ij \text{’ related to reservoir } i \text{ and its downstream } j \text{ for } j \in \tilde{\mathcal{D}}_i, i \in \mathcal{I}^V. \]

\[ M_{ijc}, R_{ijc} \text{ [MEURO], Investment cost and its periodic allocation (see below), res., required by cable } c \in C^{ij} \text{ of new transmission line } ij \in \mathcal{L}^N \text{ in the energy network.} \]
EGCTEP problem modeling. Uncertain parameters for period \( t \in T^e(g) \) in scenario group \( g \in G \)

\[ \bar{P}_{i,t,g} \quad [GWh], \text{Maximum generated energy by any power plant of candidate technology } i \in \mathcal{N} \text{ or current power plant } i \in \mathcal{I}^K. \text{ See below.} \]

\[ \bar{D}_{i,t,g} \quad [GWh], \text{Energy demand from energy network node } i \in \mathcal{I}. \]
EGCTEP problem modeling. Uncertain parameters for any period from set $\mathcal{T}^{e(g)}$ in scenario group $g \in \mathcal{G}$

$H_{i,g}$ \text{[h], Operating hours per period of any power plant of candidate technology $i \in \mathcal{N}$ or current power plant $i \in \mathcal{I}^K$.}

$\mu_g$ \text{[TM(GWh)], Upper bound on the CO}_2 \text{ that can be allowed by one GWh of energy generated by the thermal plants owned and available new plants for any 'producer' genco $\in$ GENCOS, i.e., plants in set $(\mathcal{I}^{NT})_{\text{genco}} \cup (\mathcal{I}^{KT})_{\text{genco}}$.}
EGCTEP problem modeling. Uncertain parameters for any period from set $\mathcal{T}^e(g)$ in scenario group $g \in \mathcal{G}$ (C.)

$\bar{Q}_g \ [\text{GMh}]$, Upper bound on the Green Certificates for any ’producer’ $\text{genco} \in \text{GENCOS}$, such that if they are positive the ’producer’ will be paid for.

Note: $-\bar{Q}_g$ is the related lower bound, such that in case that the Green Certificates are negative the produced will paid for.

$\gamma_g \ [-]$, Ratio to be attained of electricity generated from RES plants plus Green Certificates and total electricity generated for any ’producer’ $\text{genco} \in \text{GENCOS}$. 
EGCTEP problem modeling. Uncertain parameters for any period from set $\mathcal{T}^{e(g)}$ in scenario group $g \in \mathcal{G}$ (c.){

- $W_{i,g}$ \hspace{1cm} $[h^3/t]$, Water exogenous inflow into reservoir $i$ in any period from the set, for $i \in \mathcal{I}^V$. Note: Its value is zero for run-of-the river plants.

- $\bar{r}_{ij,g}$ \hspace{1cm} $[h^3/h]$, Water capacity for hydro power generation in candidate turbine(s) to increase the hydro power generation in 'canal' $ij$, for $j \in \tilde{D}_i$, $i \in \mathcal{I}^V$.

- $\phi_{ij,g}$ \hspace{1cm} $[\text{MWh}/h^3]$, Assumed constant converting one $h^3$ into energy generated by the identical candidate turbine(s) to increase the hydro power generation in 'canal' $ij$, for $j \in \tilde{D}_i$, $i \in \mathcal{I}^V$.

- $w_{i,g}$, $\bar{w}_{i,g}$ \hspace{1cm} $[h^3]$, Lower and upper bounds of stored water at end of any period, for $w \in \mathcal{I}^V$.

- $r_{ij,g}$, $\bar{r}_{ij,g}$ \hspace{1cm} $[h^3/t]$, Lower and upper bounds of the release water of any period from the set in 'canal' $ij$, for $j \in D_i$, $i \in \mathcal{I}^V$.}
EGCTEP problem modeling. Uncertain parameters for any period from set $\mathcal{T}^e(g)$ in scenario group $g \in \mathcal{G}$ (c.)

For each cable $c \in C^i_j$ of new transmission line $ij$, so, $ij \in \mathcal{L}^N$ in the energy network;

- $\overline{F}_{ijc,g}$ [GWh], Maximum flow allowed.
- $\lambda_{ijc,g}$ [oh], Reactance.
- $\sigma_{ijc,g}$ [\text{--}], Loss rate.
EGCTEP problem modeling. Uncertain economic parameters for period $t \in T^{e(g)}$ in scenario group $g \in G$ (c.)

$\pi^{D}_{i,t,g} \ [\text{kEURO/GWh}]$, Market electricity price at node $i \in I$.

$\pi^{GC}_{g} \ [\text{kEURO/GWh}]$, Green Certificate price.

$\pi^{CO_{2}}_{g} \ [\text{kEURO/TM}]$, CO$_2$ emission penalization.
EGCTEP problem modeling. Uncertain economic parameters for scenario group $g \in \mathcal{G}$ (c.)

$c_{n,g}^F \ [kEURO]$, Fixed power generation cost of any power plant of candidate technology $n \in \mathcal{I}^N$ at any period in the scenario group.

$c_{n,g}^V \ [kEURO/GWh]$, Variable power generation cost of any power plant of candidate technology $n \in \mathcal{I}^N$ at any period in the scenario group.

Note: Hydro power generation plants are not included.
EGCTEP problem modeling. Uncertain economic parameters for scenario group $g \in \mathcal{G}$ (c.)

$c^{F}_{i,g}$ \([kEURO]\), Fixed power generation cost of any power plant $i \in \mathcal{I}^K$ at any period in the scenario group. Note: Hydro power generation plants are not included. Note: Hydro power generation plants are not included.

$c^{V}_{i,g}$ \([kEURO/GWh]\), Variable power generation cost of current power plant $i \in \mathcal{I}^K$ at any period period in the scenario group. Note: Hydro power generation plants are not included.

$c_{ij,g}$ \([kEURO]\), Fixed power generation cost of new or current hydro power turbine(s) in 'canal' $ij$, for $j \in \mathcal{D}_i$, $i \in \mathcal{I}^V$ at period $t \in \mathcal{T}$.

$c_{ijc,g}$ \([kEURO]\), Fixed cost of cable $c \in C^{ij}$ of new transmission line $ij \in \mathcal{L}^N$ in the energy network at any period in the scenario group.
EGCTEP problem modeling. Uncertain economic parameters for period $t \in \mathcal{T}^{e(g)}$ in scenario group $g \in \mathcal{G}$ (c.)

$(B_g)_{\text{genco}}$ [MEURO], Upper bound on the total investment cost amortization that is allocated to the set of periods $\mathcal{T}^{e(g)}$ for ’producer’ genco $\in \text{GENCOS}$, due to the availability of new plants and turbines hydro power generation in scenario group $g$.

$(B_g)_{\text{tso}}$ [MEURO], Upper bound on the total investment cost amortization that is allocated to the set of periods $\mathcal{T}^{e(g)}$ for transmission system operator tso, due to the availability of the new transmission lines in scenario group $g$. 
EEECEP problem modeling. \( x \) variable for energy generation capacity expansion and \( y \) variable for transmission line expansion in the energy network at first period \( t(e(g)) \) in scenario group \( g \in G \)

The value 1 for the 0-1 variable means that the entity starts its construction at the first period of set \( T(e(g)) \) (i.e., period \( t(e(g)) \)) for scenario group \( g \in G \) and otherwise, 0.

\( x_{i,g} \) [−] for potential new power plant \( i \in I^N \).

\( x_{ij,g} \) [−] for potential new turbine(s) in ’canal’ \( ij \) for \( j \in \tilde{D}_i, i \in I^V \).

\( x_{ijc,g} \) [−] for \( c \in C^{ij} \) of new transmission line \( ij \in L^N \).

Observe that, without loss of generality, it is assumed that the starting construction period of any plant or transmission line can only be performed at the first period of the stages along the time horizon.
EGCEP problem modeling. Decision continuous variables for time period \( t \in \mathcal{T}^e(g) \) in scenario group \( g \in \mathcal{G} \)

\( p_{i,t,g} \) \([GWh]\), Energy generated by power plant \( i \in \mathcal{I}^N \cup \mathcal{I}^K \).

\( w_{i,t,g} \) \([h^3]\), Water stored in reservoir \( i \in \mathcal{I}^V \) at the beginning of the period.

\( r_{i,t,g} \) \([h^3]\), Water released in reservoir \( i \in \mathcal{I}^V \) at the period.

\( (q_{t,g})_{genco} \) \([GWh]\), Green Certificates sold \((q_{t,g} > 0)\) or bought \((q_{t,g} < 0)\) by 'producer' \( genco \in GENCOS \).

\( (o_{t,g})_{genco} \) \([TM]\), \( CO_2 \) produced in period \( t \) (to be paid for) by 'producer' \( genco \in GENCOS \).
EGCEP problem modeling. Decision continuous variables for time period $t \in \mathcal{T}^{e(g)}$ in scenario group $g \in \mathcal{G}$ (c.)

$V_{i,t,g}$ [rad], Voltage angle at node $i \in \mathcal{I}$ in the energy network.

$f_{ijc,t,g}$ [GWh] Energy flow through cable $c \in C^{ij}$ of line $(ij) \in \mathcal{L}$.

$s_{i,t,g}$ [GWh], Served energy demand from energy network node $i \in \mathcal{I}$. 
Maximizing the expected profit, Risk Neutral (RN) strategy minus the fixed generation cost of current power plants and hydro power turbines:

\[
\begin{align*}
  z = \max \sum_{\omega \in \Omega} w^\omega z^\omega - \left( \sum_{i \in I^V} \sum_{j \in D_i \setminus \tilde{D}_i} c_{ij,g} + \sum_{i \in I^K: i \leq L_i} c_{i,t}^F + \sum_{ij \in \mathcal{L}} \sum_{c \in C^{ij}} c_{ijc} \right),
\end{align*}
\]

where \( z^\omega \) gives the net present profit under scenario \( \omega \in \Omega \), subject to constraints (6)-(38).
NPV of profit $z^\omega$ under scenario $\omega \in \Omega$ is included by:
- Revenue from sale of electricity.
- Revenue from sale (or, alternatively, cost for purchase) of Green Certificates.
- Penalization of $CO_2$ emission.
- Variable generation cost of thermal power plants.
- Variable generation cost of RES power plants, without including hydro generation plants.
- Periodic debt repayment of all new power plants and new hydro power turbines.
- Fixed power generation cost of available new plants and new hydro power turbines.
- Periodic debt repayment of all new transmission lines
- Fixed transmission cost of available new transmission lines.
\[ z^\omega = \sum_{g \in A^\omega} \sum_{t \in T^{e(g)}} \frac{1}{(1 + \rho)^t} \sum_{i \in I} \pi^D_{i,t,g} s_{i,t,g} \]

\[ + \sum_{\text{genco} \in \text{GENCOS}} \left( \pi^\text{GC}_{g,t,g} \text{genco} - \pi^\text{CO}_2_{o,t,g} \text{genco} \right) \]

\[ - \sum_{i \in I^N} c^V_{n(i),g} p_{i,t,g} - \sum_{i \in I^K} c^V_{i,g} p_{i,t,g} \]

\[ - \sum_{i \in I^N} (R_i + c^F_{n(i),g}) y_1 - \sum_{i \in I^V} \sum_{j \in \bar{D}_i} (R_{ij} + c_{ij,g}) y_2 \]

\[ - \sum_{i,j \in L^N} \sum_{c \in C_{ij}} (R_{ijc} + c_{ijc,g}) y_3 \]
Equivalent 0-1 variable indicators $y_1$, $y_2$ and $y_3$ for the availability of new power plants and cables of transmission lines, res., in period $t \in T^{e(g)}$ for scenario group $g \in \mathcal{G}$

\begin{align}
    y_1 \equiv & \sum_{g' \in \mathcal{A}_g \colon t(e(g')) + S_n \leq t < t(e(g')) + S_n + L_n} x_{i,g'} \\
    y_2 \equiv & \sum_{g' \in \mathcal{A}_g \colon t(e(g')) + S_{ij} \leq t} x_{ij,g'} \\
    y_3 \equiv & \sum_{g' \in \mathcal{A}_g \colon t(e(g')) + S_{ijc} \leq t} x_{ijc,g'}
\end{align}
Defining the 0-1 character of the $x$-variable and upper bounding the number of plants of candidate technology $n \in \mathcal{N}$ that can be constructed.

\begin{align*}
  x_{i,g} & \in \{0, 1\} \quad \forall i \in \mathcal{I}^N, \ g \in \mathcal{G} \\
  \sum_{g \in \mathcal{G}} x_{i,g} & \leq 1 \quad \forall i \in \mathcal{I}^N, \ \omega \in \Omega \\
  \sum_{i \in \mathcal{I}^n} \sum_{g \in \mathcal{G}} x_{i,g} & \leq \bar{I}^n \quad \forall n \in \mathcal{N}, \ \omega \in \Omega.
\end{align*}
Electricity production bounding for current power plants

The electricity generation obtained by current power plant $i$ is lower bounded by the conditional minimum generation and upper bounded generation in period $t$.

$$P_{i,t} \leq p_{i,t,g} \leq \overline{P}_{i,t,g} \forall i \in \mathcal{I}^K, t \in \mathcal{T}^{e(g)}, g \in \mathcal{G},$$ (7)

where

$$\overline{P}_{i,t,g} = \begin{cases} \frac{1}{1000} P_i H_i,g (1 - \nu_i) & \text{if } t \leq L_i \\ 0 & \text{if } t > L_i. \end{cases}$$ (8)
Electricity production bounding for power plants of candidate technologies

The electricity generation obtained by power plant \( i \) of candidate technology \( n \in \mathcal{N} \) is lower bounded by the conditional minimum generation and upper bounded generation in period \( t \), if it is available.

\[
P_{i,t} y_1 \leq p_{i,t,g} \leq P_{n(i),t,g} y_1 \forall i \in \mathcal{I}^N, t \in \mathcal{T}^{e(g)}, g \in \mathcal{G},
\]

where

\[
y_1 \equiv \sum_{g' \in \mathcal{A}_g : t(e(g')) + S_n \leq t < t(e(g')) + S_n + L_n} x_{i,g'}
\]

The maximum electricity production \( P_{n,t,g} \) of a power plant of technology \( n \in \mathcal{N} \) at period \( t \) in group \( g \) is defined as

\[
P_{n,t,g} = \frac{1}{1000} P_n H_{n,g} (1 - \nu_n).
\]

Note: The parameters \( H_{n,g} \) and \( H_{i,g} \) take into account possible plant breakdown and maintenance.
Defining the 0-1 character of the $x$-variable and upper bounding the number of new turbines in the hydro valleys that can be constructed

$$x_{ij,g} \in \{0, 1\} \quad \forall j \in \tilde{D}_i, \ i \in I^V, \ g \in G$$

$$\sum_{g \in A^\omega} x_{ij,g} \leq 1 \quad \forall j \in \tilde{D}_i, \ i \in I^V, \ \omega \in \Omega.$$ (12)
Defining the replicated network of the Hydro valleys along the periods of the time horizon:

\[
W_{i,t,g} + \sum_{j \in U_i} W_{i,j} = \sum_{j \in D_i} r_{ij,t,j} + W_{i,t+1,\hat{g}}
\]

\[
\forall i \in \mathcal{I}^V, t \in \mathcal{T}^e(g), g \in \mathcal{G}, \quad (13)
\]

where \( \hat{g} = g \) for \( t + 1 \in \mathcal{T}^e(g) \) and otherwise, \( \hat{g} = g' \), being \( g' \in \mathcal{G}^e \), such that \( t + 1 = t(e) \).

\[
\underline{W}_{i,t,g} \leq W_{i,t,g} \leq \overline{W}_{i,t,g} \quad \forall i \in \mathcal{I}^V, t \in \mathcal{T}^e(g), g \in \mathcal{G} \quad (14)
\]

\[
\underline{r}_{ij,g} \leq r_{ij,t,g} \leq \overline{r}_{ij,g} \quad \forall j \in \mathcal{D}_i, i \in \mathcal{I}^V, t \in \mathcal{T}^e(g), g \in \mathcal{G}. \quad (15)
\]
Electricity production from current and potential new turbines in the hydro valleys

\[
p_{i,t,g} = \sum_{j \in D_i \setminus \tilde{D}_i} \frac{1}{1000} (\phi_{ij}^K \min\{r_{ij,t,g}, \bar{T}_{ij}^K\})(1 - y_2) + \\
\sum_{j \in \tilde{D}_i} \frac{1}{1000} (\phi_{ij,g}^N \min\{r_{ij,t,g}, \bar{T}_{ij,g}^N\})y_2
\]  

(16)

\[
\forall i \in \mathcal{I}^V, \ t \in \mathcal{T}^{e(g)}, \ g \in \mathcal{G},
\]  

(17)

where

\[
y_2 \equiv \sum_{g' \in A_g : t(e(g')) + s_{ij} \leq t} x_{ij,g'}.
\]  

(18)
Periodic investment cost allocation of new power plants of 'producer' \( genco \in GENCOS \) for the set of periods \( T^e(g) \) in scenario group \( g \in G \)

\[
\left( \sum_{t \in T^e(g)} \frac{1}{(1 + \rho)^t} \right) \sum_{i \in (I^N)_{genco}} R_i y_1 + \sum_{i \in (I^V)_{genco}} \sum_{j \in \tilde{D}_i} R_{ij} y_2 \leq (B_g)_{genco},
\]

where

\[
y_1 \equiv \sum_{g' \in A_g : t(e(g')) + S_n \leq t < t(e(g')) + S_n + L_n} x_{i,g'}
\]

\[
y_2 \equiv \sum_{g' \in A_g : t(e(g')) + S_{ij} \leq t} x_{ij,g'}
\]
Periodic investment cost allocation of new power plants of ’producer’ \textit{genco} \in GENCOS for the set of periods $\mathcal{T}^{e(g)}$ in scenario group $g \in \mathcal{G}$ (c.)

and $R_i$ and $R_{ij}$ are the debt repayment per period for investment in power plant $i$ of candidate technology $n(i)$ and new turbine(s) for ’canal $ij$ of the hydro power generation, such that

$$R_i = \frac{1000 \cdot M_i \cdot P_n(i) \cdot \rho}{1 - (\frac{1}{1+\rho})^L n(i)}$$

$$R_{ij} = \frac{1000 \cdot M_{ij} \cdot \rho}{1 - (\frac{1}{1+\rho})^L ij}$$

(21)
The amount of electricity $q_{t,g}$, for which in period $t$ under scenario group $g$ the corresponding Green Certificates are bought by 'producer' $\text{genco} \in \text{GENCOS}$ if $q_{t,g} < 0$, or sold if $q_{t,g} > 0$ is defined as follows,

$\left( q_{t,g} \right)_{\text{genco}} = \sum_{i \in (\mathcal{I}^V)_{\text{genco}} \cup \mathcal{I}^{NR}_{\text{genco}} \cup (\mathcal{I}^R)_{\text{genco}}} p_{i,t,g} - \gamma t \sum_{i \in (\mathcal{I})_{\text{genco}}} p_{i,t,g}$ \hspace{1cm} (22) 

$-Q_g \leq \left( q_{t,g} \right)_{\text{genco}} \leq Q_g \hspace{1cm} \forall t \in \mathcal{T}^{e(g)}, \ g \in \mathcal{G}. \hspace{1cm} (23)$
CO2 emission definition and bounding

The amount $\left( o_{t,g} \right)_{\text{genco}}$ of CO2 that can be emitted (end, being paid by 'producer' genco $\in$ GENCOS in period $t$ under scenario group $g$ is defined as follows,

$$
\left( o_{t,g} \right)_{\text{genco}} = \sum_{i \in (I^N T)_{\text{genco}}} \zeta_n(i) p_{i,t,g} + \sum_{i \in (I^K T)_{\text{genco}}} \zeta_i p_{i,t,g}
$$

$\forall t \in T^{e(g)}, g \in \mathcal{G}$.  

(24)

$$
0 \leq \left( o_{t,g} \right)_{\text{genco}} \leq \mu_g \sum_{i \in (I^N T)_{\text{genco}} \cup (I^K T)_{\text{genco}}} p_{i,t,g} \quad \forall t \in T^{e(g)}, g \in \mathcal{G}.
$$

(25)
Defining the 0-1 $y$-variable

$x_{ijc,g} \in \{0, 1\} \quad \forall c \in C^{ij}, \ ij \in L^N, \ g \in G$

$\sum_{g \in A^\omega} x_{ijc,g} \leq 1 \quad \forall c \in C^{ij}, \ ij \in L^N, \ \omega \in \Omega.$

(26)
First Kirchhoff law balancing power in the nodes of the transmission network

Energy generation nodes:

\[
\sum_{j:(j,i) \in \mathcal{L}} \sum_{c \in \mathcal{C}^j} \sigma_{ijc,g} f_{ijc,t,g} + p_{i,t,g} - \sum_{j:(i,j) \in \mathcal{L}} \sum_{c \in \mathcal{C}^j} \sigma_{ijc,g} f_{ijc,t,g} + s_{i,t,g} = D_{i,t,g} \quad \forall t \in T^{e(g)}, \ g \in \mathcal{G}, \ i \in \mathcal{I}^K \cup \mathcal{I}^N. \tag{27}
\]

Energy transmission nodes:

\[
\sum_{j:(j,i) \in \mathcal{L}} \sum_{c \in \mathcal{C}^j} \sigma_{ijc,g} f_{ijc,t,g} - \sum_{j:(i,j) \in \mathcal{L}} \sum_{c \in \mathcal{C}^j} \sigma_{ijc,g} f_{ijc,t,g} + s_{i,t,g} = D_{i,t,g} \quad \forall t \in T^{e(g)}, \ g \in \mathcal{G}, \ i \in \mathcal{I}^{Tr}. \tag{28}
\]
Second Kirchhoff law (Voltage law) defining energy flow in the transmission network

Current energy transmission network:

\[ f_{ijc,t,g} = \frac{V_{i,t,g} - V_{j,t,g}}{\lambda_{ijc}^K} \quad \forall t \in \mathcal{T}^e(g), \; g \in G, \; c \in C^{ij}, \; ij \in L^K \]  \hfill (29)

Expansion energy transmission network:

\[-M(1 - y_3) \leq f_{ijc,t,g} - \frac{V_{i,t,g} - V_{j,t,g}}{\lambda_{ijc}^K} \leq M(1 - y_3) \]
\[ \forall t \in \mathcal{T}^e(g), \; g \in G, \; c \in C^{ij}, \; ij \in L^N \]  \hfill (30)

where

\[ y_3 \equiv \sum_{g' \in A_g : t(e(g')) + S_{ijc} \leq t} x_{ijc,g'} \]  \hfill (31)
Transmission network variables defining

\( s_{i,t,g} \in \mathbb{R}^+ \quad \forall t \in T^e(g), \ g \in G, \ i \in I \) \hspace{1cm} (32)

\[-F_{ijc,g} \leq f_{ijc,t,g} \leq F_{ijc,g} \quad \forall t \in T^e(g), \ g \in G, \ c \in C^i, \ ij \in L^K \] \hspace{1cm} (33)

\[-F_{ijc,g}y_3 \leq f_{ijc,t,g} \leq F_{ijc,g}y_3 \quad \forall t \in T^e(g), \ g \in G, \ c \in C^i, \ ij \in L^N] \hspace{1cm} (34)

where

\[ y_3 \equiv \sum_{g' \in A_g : t(e(g')) + s_{ijc} \leq t} X_{ijc,g'} \] \hspace{1cm} (35)
Periodic investment cost allocation of cables of new transmission lines of \( tso \) in set \( TSOS \) for the set of periods \( T_\epsilon^{e(g)} \) in scenario group \( g \in G \)

\[
\left( \sum_{t \in T_\epsilon^{e(g)}} \frac{1}{(1 + \rho)^t} \right) \sum_{ij \in (L^N)_tso} \sum_{c \in C^{ij}} R_{ijc} y_3 \leq (B_g)_{tso}
\]  

(36)

where

\[
y_3 \equiv \sum_{g' \in A_g : t(e(g')) + S_{ijc} \leq t} x_{ijc, g'}
\]  

(37)

and \( R_{ijc} \) is the debt repayment per period for investment in cable \( c \in C^{ij} \) of transmission line \( ij \), such that

\[
R_{ijc} = \frac{1000 \cdot M_{ijc} \cdot \rho}{1 - \left( \frac{1}{1 + \rho} \right)^{L_{ijc}}}
\]  

(38)
Time Stochastic Dominance (TSD) risk averse strategy. Motivation

- The risk neutral (RN) model maximizes the objective function expected value. It ignores the variability of the objective function value over the scenarios, in particular the “left” tail of the non-wanted scenarios.
- It only considers the principal function-criterion.
- There are some risk averse approaches that additionally deal with risk management; among them, TSD reduces the risk of the negative impact of the solutions in non-wanted scenarios in a better way than the others under some circumstances.
The risk neutral (RN) model maximizes the objective function expected value. It ignores the variability of the objective function value over the scenarios, in particular the “left” tail of the non-wanted scenarios.

- It only considers the principal function-criterion.
- There are some risk averse approaches that additionally deal with risk management; among them, TSD reduces the risk of the negative impact of the solutions in non-wanted scenarios in a better way than the others under some circumstances.
The risk neutral (RN) model maximizes the objective function expected value. It ignores the variability of the objective function value over the scenarios, in particular the “left” tail of the non-wanted scenarios.

It only considers the principal function-criterion.

There are some risk averse approaches that additionally deal with risk management; among them, TSD reduces the risk of the negative impact of the solutions in non-wanted scenarios in a better way than the others under some circumstances.
Time Stochastic Dominance (TSD) risk averse strategy. Motivation

- The risk neutral (RN) model maximizes the objective function expected value. It ignores the variability of the objective function value over the scenarios, in particular the “left” tail of the non-wanted scenarios.

- It only considers the principal function-criterion.

- There are some risk averse approaches that additionally deal with risk management; among them, TSD reduces the risk of the negative impact of the solutions in non-wanted scenarios in a better way than the others under some circumstances.
Time Stochastic Dominance (TSD) risk averse strategy. Motivation

- The risk neutral (RN) model maximizes the objective function expected value. It ignores the variability of the objective function value over the scenarios, in particular the “left” tail of the non-wanted scenarios.
- It only considers the principal function-criterion.
- There are some risk averse approaches that additionally deal with risk management; among them, TSD reduces the risk of the negative impact of the solutions in non-wanted scenarios in a better way than the others under some circumstances.
The strategy also aims to maximize the objective function expected value as RN, but, additionally, a modeler-driven set of given thresholds on the value of the given function-criteria for each scenario group (or scenario) should be satisfied with a bound \textit{target} on the deficit (shortfall) on reaching each threshold, a bound \textit{target} on the probability of having deficit and a bound \textit{target} on the expected deficit.

We will model the risk averse TSD strategy for multistage stochastic mixed 0-1 programs at the price of including some new variables and constraints, as a mixture of the FSD: first- (Gollmer-Neise-Schultz SIOPT’08), and SSD: second order- (Gollmer-Gotzes-Schultz MP’11) stochastic dominance constraints induced by integer-linear recourse for two-stage
The strategy also aims to maximize the objective function expected value as RN, but, additionally, a modeler-driven set of given thresholds on the value of the given function-criteria for each scenario group (or scenario) should be satisfied with a bound \( target \) on the deficit (shortfall) on reaching each threshold, a bound \( target \) on the probability of having deficit and a bound \( target \) on the expected deficit.

We will model the risk averse TSD strategy for multistage stochastic mixed 0-1 programs at the price of including some new variables and constraints, as a mixture of the FSD: first- (Gollmer-Neise-Schultz SIOPT'08), and SSD: second order- (Gollmer-Gotzes-Schultz MP'11) stochastic dominance constraints induced by integer-linear recourse for two-stage.
The strategy also aims to maximize the objective function expected value as RN, but, additionally, a modeler-driven set of given thresholds on the value of the given function-criteria for each scenario group (or scenario) should be satisfied with a bound *target* on the deficit (shortfall) on reaching each threshold, a bound *target* on the probability of having deficit and a bound *target* on the expected deficit.

We will model the risk averse TSD strategy for multistage stochastic mixed 0-1 programs at the price of including some new variables and constraints, as a mixture of the FSD:*first*- (Gollmer-Neise-Schultz SIOPT’08), and SSD:*second order*- (Gollmer-Gotzes-Schultz MP’11) stochastic dominance constraints induced by integer-linear recourse for two-stage
TSD risk averse strategy:  
Set of modeler-driven functions and profiles

- $\mathcal{F}$, set of function-criteria to be satisfied, such that function $f = 1$ is the objective function to maximize.
- $\mathcal{E}^f \subseteq \mathcal{E}$, set of stages where **Time Stochastic Dominance (TSD)** has to be considered for function-criterion $f \in \mathcal{F}$.
- Set of profiles, say $\mathcal{P}^e$ for $e \in \mathcal{E}^f$, $f \in \mathcal{F}$,
- For each profile $p \in \mathcal{P}^e$ in TSD stage $e \in \mathcal{E}^f$ for function-criterion $f \in \mathcal{F}$:
  - $\phi^p$, objective function threshold to be satisfied up to last period in any scenario group $g$ of stage $e$.
  - $D^p$, upper bound *target* on the deficit (shortfall) that is allowed on reaching threshold $p$ up to the last period in any scenario group $g$ of stage $e$, for $g \in \mathcal{G}^e$.
  - $\alpha^p$, upper bound *target* on the expected deficit on reaching threshold $\phi^p$.
  - $\beta^p$, upper bound *target* on the fraction of scenarios with deficit on reaching threshold $\phi^p$.  

Laureano F. Escudero  
Universidad Rey Juan Carlos, Móstoles (M)  
EGTCEP
For each pair scenario group $g$ and profile $p$ in TSD stage $e$ and function-criterion $f$, for $g \in G^e$, $p \in P^e$, $e \in E^f$, $f \in F$:

- $d^{g,p}$, deficit (shortfall) continuous variable that, obviously, is equal to the difference (if it is positive) between threshold $\phi^p$ and the value of function-criterion $f$ up to last period of scenario group $g$.

- $\nu^{g,p}$, 0-1 variable such that its value is 1 if $d^{g,p} > 0$ and otherwise, 0.
TSD risk averse strategy: Model

\[ z_{TSD} = \text{máx} \sum_{g \in G} \sum_{t \in \mathcal{T}^e(g)} w^g(a^t,g x^t,g + b^t,g y^t,g) \]

\[ - \sum_{f \in \mathcal{F}} \sum_{e \in \mathcal{E}^f} \sum_{p \in \mathcal{P}^e} (M^p_D \varepsilon^p_D + M^p_\alpha \varepsilon^p_\alpha + M^p_\beta \varepsilon^p_\beta) \]  \hspace{1cm} (39)

subject to\]

\[ \sum_{g' \in \mathcal{A}^g} \sum_{t' \in \mathcal{T}^e(g') : t' \leq t} (A^t',g' x^{t'},g' + B^t',g' y^{t'},g') = h^t,g \quad \forall t \in \mathcal{T}^e(g), \; g \in G \]

\[ x^{t,g} \in \{0, 1\}^{n_x(t,g)}, \; y^{t,g} \in \mathbb{R}^{n_y(t,g)} \quad \forall t \in \mathcal{T}^e(g), \; g \in G \]

and
TSD risk averse strategy: Model (c.)

\[ \sum_{g' \in A^g} \sum_{t' \in T^e(g')} (a_{t'^g} x_{t'^g} + b_{t'^g} y_{t'^g}) + d_{g,p} \geq \phi_p \]

\[ \forall g \in G^e, p \in P^e, e \in E^f, f \in F \]

\[ d_{g,p} \leq D^p \nu_{g,p} + \epsilon_D^p, \nu_{g,p} \in \{0, 1\} \quad \forall g \in G^e, p \in P^e, e \in E^f, f \in F \]

\[ \sum_{g \in G^e} w_{g} d_{g,p} \leq \alpha_p + \epsilon_\alpha^p \quad \forall p \in P^e, e \in E^f, f \in F \]

\[ \sum_{g \in G^e} w_{g} \nu_{g,p} \leq \beta_p + \epsilon_\beta^p \quad \forall p \in P^e, e \in E^f, f \in F. \]
Risk Averse multistage Stochastic Dominance Constraints (SDC) strategies: References

- First-order SDC: Gollmer-Neise-chultz SIOPT’08 for two-stage.
- Second-order SDC: Gollmer-Gotzes-Schultz MP’11 for two-stage.
Multistage decomposition methods

- **Lagrangeans** (MCLD strong lower bound provider), LFE-Garín-Unzueta COR12 and submitted 2014

- **Branch-and-Fix Coordination (BFC):**
  - exact sequential BFC risk neutral
    (LFE-Garín-Merino-Pérez COR’12)
  - exact parallel computing BFC risk neutral
    (Aldasoro-LFE-Merino-Pérez COR’13)
  - inexact ELP risk neutral
    (Beltrán-Royo-LFE-Monge-Rodriguez.Revines COR’14)
  - Parallel computing SDP risk neutral
    (Aldasoro-LFE-Merino-Monge-Pérez submitted 2014)
  - plus treating the cross scenario group constraints:
    - exact BFC risk averse TSD (LFE-Garín-Merino-Pérez
      submitted 2014)
    - inexact SDP risk averse TSD
      (LFE-Monge-Romero.Morales submitted 2014)
    - inexact FRC risk averse TSD
      (LFE-Garín-Pizarro-Unzueta
      in preparation)
Successful results: Production planning

Computational Characteristics:

- MPI, Message Passing Interface.
- CPLEX v12.5
- Experimental Parallel-SDP code in C (Aldasoro-LFE-Merino-Perez COR13).
- ARINA computational cluster, SGI/IZO-SGIker at UPV/EHU,

We have used 8 xeon nodes, where each has 12 processors and 48Gb of RAM, 2.4GHz (96 total processors).

- Instances from Cristobal-LFE-Monge COR09.
Summary. Instance c64

- $T=16$ periods, $E=3$ stages, Randomly generated $\Omega=7766$ scenarios
- $m=4.25$ Million cons, $n01=1.16$ Million 0-1 varis, $nc=2.7$ Million continuous vars
- S-SDP: 22167 secs, number of $nprob=3249$ MIP subproblems
- P-SDP: $GG = 0.16\%$ optimality gap versus plain use CPLEX v.12.5, elapsed time=3442 secs, Efficiency=53.67\%
- CPLEX: running out of memory (35Gb) after 5926 secs, solution value with OG=1.80 $\%$ quasi-optimality gap at stopping instant time.
Summary. Instance c85

- $T=16$ periods, $E=4$ stages, Randomly generated $\Omega=15435$ scenarios
- $m=57.8$ million cons, $n01=15.4$ million 0-1 vars, $nc=38.5$ million continuous vars
- S-SDP: 26180 secs, number of $nprob=517$ MIP subproblems.
- P-SDP: 2446 secs, Efficiency=89.19.
- CPLEX: Stop due to out of memory (35Gb), no LP feasible solution at 3003 secs.
BIBLIOGRAPHY AND REFERENCES


L.F. Escudero. On energy generators maintenance scheduling constrained by the hourly distribution of the weekly energy demand, Report G320 3420 IBM Scientific Center, Palo Alto (CA, USA), 1981.


EUROSTAT.epp.eurostat.ec.europa.eu/portal/page/portal/eurostat/home/


NSCOG. www.The North Seas Countries’ Offshore Grid Initiative.


