A new primal-dual framework for European day-ahead electricity auctions

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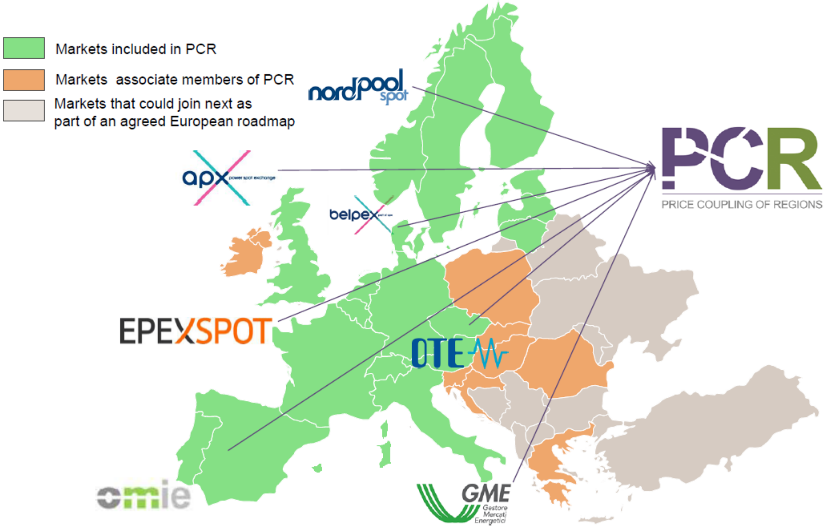
Towards the Single European Market: Next Steps



http://www.epexspot.com/en/market-coupling/pcr

1. Background information
   - Day-ahead Electricity Markets
   - Equilibrium with block orders ? : Examples
   - Primal Program (welfare maximizing dispatch)
   - Duality and Linear Equilibrium Prices
2. Get rid of the subset of compl. constr. ? Three ways
   - Linearise each of them...
   - Equality of objective functions + linearisation of quadratic terms
   - New primal-dual framework ...
3. Algorithmic application - Strengthened (locally valid) Benders cuts
4. Minimizing opportunity costs, Maximizing the traded volume, etc
5. Numerical results
   - Maximizing welfare, new approach
   - Opportunity costs vs Welfare
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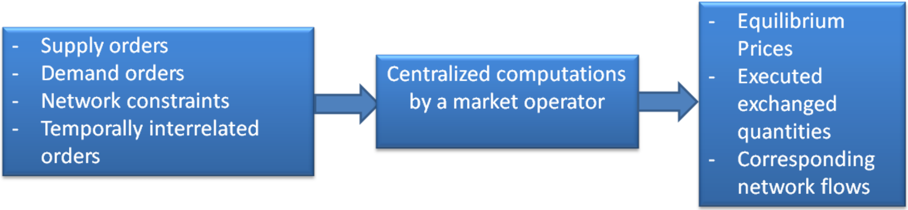
Introduction

Day-ahead Markets:

- 24 periods (23 or 25 once a year)
- several areas/locations for bids + network constraints
- Both demand and offer bids (elastic demand)

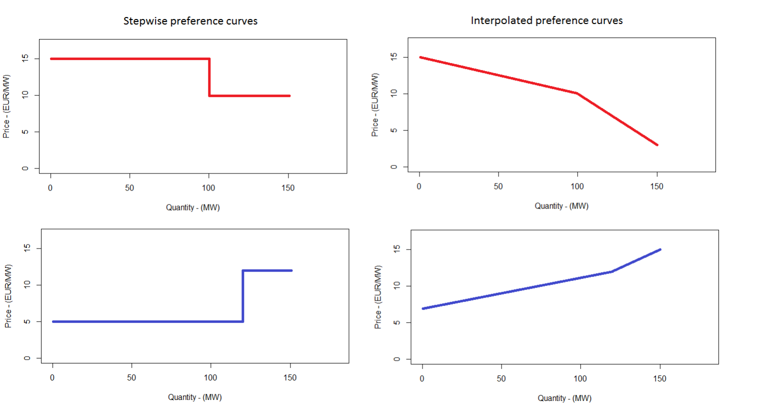


We are interested in uniform/linear prices.

Here: only one location *but everything holds for any linear transmission network representation (Flow-based, ATC, etc)*

Main kinds of bids - CWE region (EPEX, APX), Nordpool - :

Hourly bids - (Monotonic) DEMAND / SUPPLY bid curves



Block bids, i.e. 'binary bids'

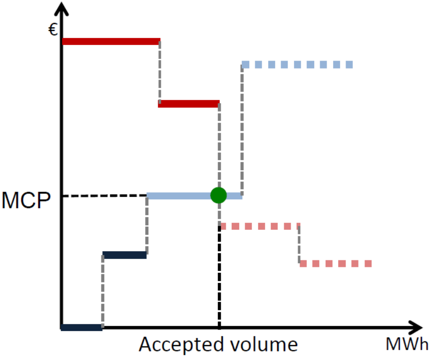
Span multiple periods and "fill-or-kill condition": bid must be entirely accepted or rejected.

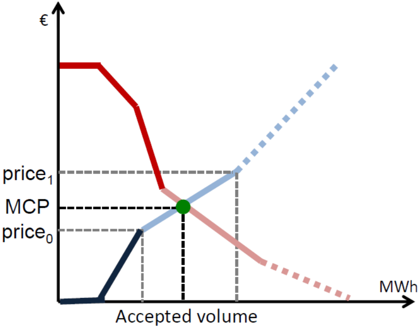
Well-behaved, continuous setting:

Market Equilibrium : *∃ MCP such that :*

- **in-the-money bids -> accepted**
- **out-of-the-money bids -> rejected**
- **at-the-money bids -> accepted / frac. / rejected**
  for piecewise lin. bids, fraction accepted w.r.t to p0 < MCP < p1

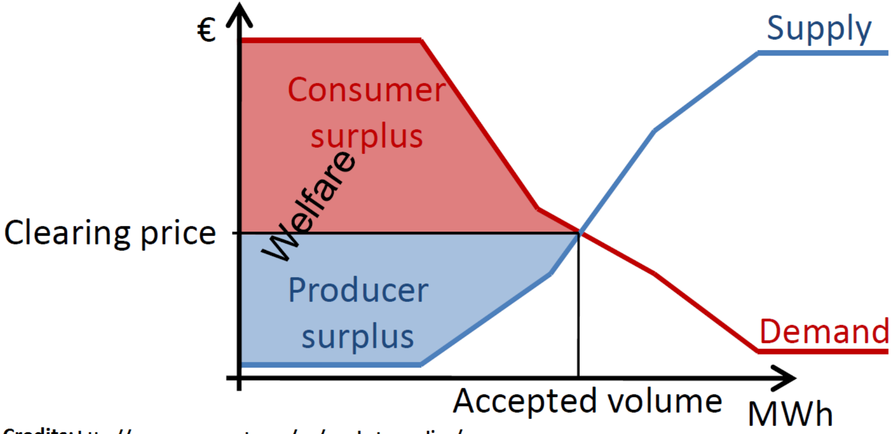
Demand in-the-money
Demand out-of-the-money
Supply in-the-money
Supply at-the-money
Supply out-of-the-money





Credits: **http://www.epexspot.com/en/market-coupling/pcr**

Well-behaved convex context (convexity of feas. set & welfare function):
Market equilibrium $\leftrightarrow$ Welfare maximization
(True more generally when considering spatially separated markets, and remarkable according to Paul Samuelson in [a])



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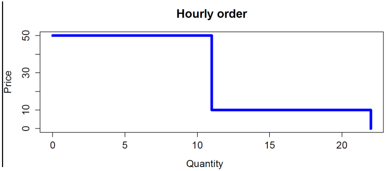
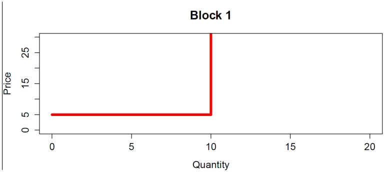
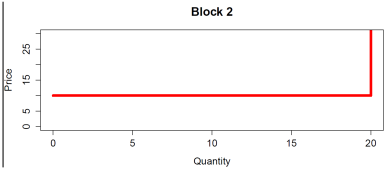
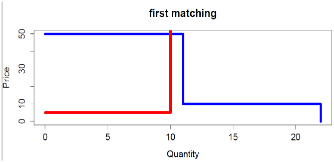
Equilibrium with block orders ?...



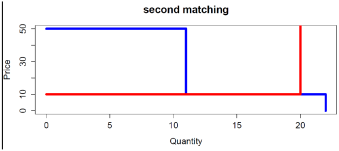






price = 50,
welfare = 450

traded volume = 10
opportunity costs = 800 of block 2

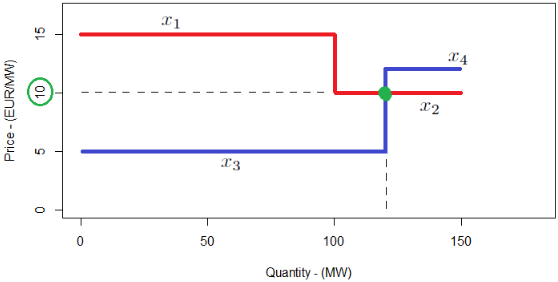


price = 10
welfare = 11 x 40 = 440

traded volume = 20
opportunity costs = 50 of block 1

Welfare maximizing solution $\neq$ Traded volume maximizing solution
Welfare maximizing solution $\neq$ Opportunity costs minimizing solution

Matching *in the convex case* as a simple LP:



$$\max_{x_i \geq 0} \quad (100 \times 15)x_1 + (50 \times 10)x_2 - (120 \times 5)x_3 - (30 \times 12)x_4$$

$$s.t. \quad x_i \leq 1 \qquad \forall i \in \{1,2,3,4\} \quad [s_i] \qquad (1)$$

$$100x_1 + 50x_2 = 120x_3 + 30x_4 \qquad [p_m] \qquad (2)$$

welfare optimal solution $= 1100$ with $x_1 = 1, x_2 = \frac{2}{5}, x_3 = 1, x_4 = 0$

p_m gives the equilibrium market clearing price

Primal (welfare maximizing) program:

$$\max_{x_i, y_j} \quad \sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j$$

subject to:

$$x_i \leq 1 \qquad \forall i \in I \quad \text{"}[s_i]\text{"} \tag{3}$$

$$y_j \leq 1 \qquad \forall j \in J \quad \text{"}[s_j]\text{"} \tag{4}$$

$$\sum_i Q^i x_i + \sum_j Q^j y_j = 0 \qquad \text{"}[p_m]\text{"} \tag{5}$$

$$x_i, y_j \geq 0, \quad y_j \in \mathbb{Z}. \tag{6}$$

$Q < 0$ for sell orders and $Q > 0$ for buy orders !

Classical MPCC formulation Of European market rules

$$\max_{x_i, y_j, p_m, s_i, s_j} \sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j$$

subject to:

$$x_i \leq 1 \qquad \forall i \in I \quad [s_i]$$

$$y_j \leq 1 \qquad \forall j \in J \quad "[s_j]"$$

$$\sum_i Q^i x_i + \sum_j Q^j y_j = 0 \qquad [p_m]$$

$$x_i, y_j \geq 0, \quad \boxed{y_j \in \mathbb{Z}}$$

Primal constraints (feasible dispatch)

$$s_i + Q^i p_m \geq Q^i P^i \qquad \forall i \in I \quad [x_i]$$

$$\boxed{s_j + Q^j p_m \geq Q^j P^j} \qquad \forall j \in J \quad [y_j]$$

$$s_i, s_j \geq 0$$

Dual program (prices)

$$s_i(1 - x_i) = 0 \qquad \forall i \in I$$

$$\cancel{s_j(1 - y_j) = 0} \qquad \forall j \in J$$

$$x_i(s_i + Q^i p_m - Q^i P^i) = 0 \qquad \forall i \in I$$

$$y_j(s_j + Q^j p_m - Q^j P^j) = 0 \qquad \forall j \in J$$

Related compl. constraints

Walrasian equilibrium

Paradoxically rejected block orders allowed

Classical MPCC formulation
Of European market rules

$$\max_{x_i, y_j, p_m, s_i, s_j} \quad \sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j$$

subject to:

(i) Equilibrium for hourly orders

(ii) optimality conditions for fixed values of 'y' (block order selection)

$$\begin{aligned}
& x_i \leq 1 && \forall i \in I && [s_i] \\
& y_j \leq 1 && \forall j \in J && "[s_j]" \\
& \sum_i Q^i x_i + \sum_j Q^j y_j = 0 && && [p_m] \\
& x_i, y_j \geq 0, \ \boxed{y_j \in \mathbb{Z}} && &&
\end{aligned}$$

Primal constraints (feasible dispatch)

$$\begin{aligned}
& s_i + Q^i p_m \geq Q^i P^i && \forall i \in I && [x_i] \\
& s_j + Q^j p_m \geq Q^j P^j && \forall j \in J && [y_j] \\
& s_i, s_j \geq 0 && &&
\end{aligned}$$

Dual program (prices)

$$\begin{aligned}
& s_i(1 - x_i) = 0 && \forall i \in I \\
& \cancel{s_j(1 - y_j) = 0} && \forall j \in J \\
& x_i(s_i + Q^i p_m - Q^i P^i) = 0 && \forall i \in I \\
& y_j(s_j + Q^j p_m - Q^j P^j) = 0 && \forall j \in J
\end{aligned}$$

Related compl. constraints

Market Equilibrium

Paradoxically rejected block orders allowed

Primal (welfare maximizing) program with blocks fixed:

$$\max_{x_i, y_j} \quad \sum_i Q^i P^i x_i + \cancel{\sum_j Q^j P^j y_j}$$

subject to:

$$x_i \leq 1 \qquad \forall i \in I \quad [s_i]$$

$$\sum_i Q^i x_i = -\sum_j Q^j y_j \qquad [p_m]$$

$$x_i \geq 0,$$

Dual with these blocks fixed:

$$\min_{s_i, p_m} \sum_i s_i - p_m\left(\sum_j Q^j y_j\right)$$

subject to:

$$s_i + Q^i p_m \geq Q^i P^i \ \forall i \in I \quad [x_i]$$

$$s_i \geq 0$$

Complementarity Constraints:

$$s_i(1 - x_i) = 0 \qquad \forall i \in I$$

$$x_i(s_i + Q^i p_m - Q^i P^i) = 0 \qquad \forall i \in I$$

Classical MPCC formulation
Of European market rules

$$\max_{x_i, y_j, p_m, s_i, s_j} \sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j$$

subject to:

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(ii) optimality conditions for fixed values of 'y' (block order selection)

$$x_i \leq 1 \qquad \forall i \in I \quad [s_i]$$

$$y_j \leq 1 \qquad \forall j \in J \quad "[s_j]"$$

$$\sum_i Q^i x_i + \sum_j Q^j y_j = 0 \qquad [p_m]$$

$$x_i, y_j \geq 0, \ \boxed{y_j \in \mathbb{Z}}$$

Primal constraints (feasible dispatch)

$$s_i + Q^i p_m \geq Q^i P^i \qquad \forall i \in I \quad [x_i]$$

$$s_j + Q^j p_m \geq Q^j P^j \qquad \forall j \in J \quad [y_j]$$

$$s_i, s_j \geq 0$$

Dual program (prices)

$$s_i(1 - x_i) = 0 \qquad \forall i \in I$$

$$\cancel{s_j(1 - y_j) = 0} \qquad \forall j \in J$$

$$x_i(s_i + Q^i p_m - Q^i P^i) = 0 \qquad \forall i \in I$$

$$y_j(s_j + Q^j p_m - Q^j P^j) = 0 \qquad \forall j \in J$$

Related compl. constraints

Market Equilibrium

Paradoxically rejected block orders allowed

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First way: linearise all three kinds of compl. constraints

real instances: more than 50 000 hourly bids $\rightarrow$ 100 000 aux. bin. vars. !!
Not tractable for real large-scale instances !

$$s_i(1 - x_i) = 0$$
$$x_i(s_i + Q^i p_m - Q^i P^i) = 0$$
$$y_j(s_j + Q^j p_m - Q^j P^j) = 0$$

With big-M's and $z_i, v_i \in \{0, 1\}$, i.e. $2|\textit{Hourly bids}|$ auxiliary bin. vars.:

- $0 \leq (1 - x_i) \leq M z_i$
- $0 \leq s_i \leq M(1 - z_i)$
- $0 \leq x_i \leq M v_i$
- $0 \leq (s_i + Q^i p_m - Q^i P^i) \leq M(1 - v_i)$
- $0 \leq s_j + Q^j p_m - Q^j P^j \leq M(1 - y_j)$ (since $y_j \in \{0, 1\}$)

Second way: linearise quad. terms in equality of objectives

Proposition of Zak. et al. [c]

Price-based decisions (cond. on block order select.)

$$\max_{x_i, y_j, p_m, s_i, s_j} \sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j$$

subject to:

(i) Equilibrium for hourly orders

(ii) optimality conditions for fixed values of 'y' (block order selection)

$$x_i \leq 1 \qquad \forall i \in I \quad [s_i]$$

$$y_j \leq 1 \qquad \forall j \in J \quad "[s_j]"$$

$$\sum_i Q^i x_i + \sum_j Q^j y_j = 0 \qquad [p_m]$$

$$x_i, y_j \geq 0, \quad \boxed{y_j \in \mathbb{Z}}$$

Primal constraints (feasible dispatch)

$$s_i + Q^i p_m \geq Q^i P^i \qquad \forall i \in I \quad [x_i]$$

$$\boxed{s_j + Q^j p_m \geq Q^j P^j} \qquad \forall j \in J \quad [y_j]$$

$$s_i, s_j \geq 0$$

Dual program (prices)

$$\sum_i Q^i P^i x_i = \sum_i s_i - p_m\left(\sum_j Q^j y_j\right)$$

$$0 \leq s_j + Q^j p_m - Q^j P^j \leq M(1 - y_j)$$

Paradoxically rejected block orders allowed

Second way: linearise quad. terms in equality of objectives

Proposition of Zak. et al. [c]

Again, not tractable for real large-scale instances (cf. paper)

Well-known trick:
To linearise $p_m y_j$, replace it by z_j, adding constraints with big-Ms:

$$
\begin{aligned}
z_j &\leq M y_j \\
z_j &\leq p_m \\
z_j &\geq p_m - M(1 - y_j)
\end{aligned}
$$

If n coupled markets, n prices, and $n \times |block\ orders|$ aux. vars.
e.g in the CWE region: 24 periods, 4 counties $\rightarrow$ 96 submarkets
and about 700 block orders $\rightarrow$ 67200 auxiliary vars.

Third way: yields a useful primal-dual framework

Primal-dual framework:

Feasible Set *LMM* defined by:

$$
\begin{array}{ll}
x_i \leq 1 & \forall i \in I \\
y_j \leq 1 & \forall j \in J \\
\sum_i Q^i x_i + \sum_j Q^j y_j = 0 & \\
x_i, y_j \geq 0, \quad y_j \in \mathbb{Z} &
\end{array}
$$

$$
\begin{array}{lll}
s_i + Q^i p_m \geq Q^i P^i & & \forall i \in I \\
s_j + d_{0_j} - \cancel{d_{1_j}} + Q^j p_m \geq Q^j P^j & & \forall j \in J \\
d_{0_j} \leq M_j(1 - y_j) & d_{0_j} \text{ *upper bound* on the opportunity cost of order } j & \forall j \in J \\
\cancel{d_{1_j} \leq M_j y_j} & d_{1_j} \text{ *upper bound* on the actual loss of executed order j} & \forall j \in J \\
s_i, s_j, d_{0_j}, d_{1_j} \geq 0, \quad param.: \ M_j >> 0 & &
\end{array}
$$

$$
\sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j \geq^{\cdots=} \sum_i s_i + \sum_j s_j - \cancel{\sum_{j_1 \in J_1} d_{1_{j_1}}}
$$

Strong duality <-> 'relaxed complementarity constraints'

Core European market model, new formulation:

No auxiliary variables at all & *tractable for large-scale instances*

$$\max_{x_i, y_j, p_m, s_i, s_j} \quad \sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j$$

subject to:

$$x_i \leq 1 \qquad \forall i \in I \quad [s_i] \tag{7}$$

$$y_j \leq 1 \qquad \forall j \in J \quad "[s_j]" \tag{8}$$

$$\sum_i Q^i x_i + \sum_j Q^j y_j = 0 \qquad [p_m] \tag{9}$$

$$x_i, y_j \geq 0, \quad y_j \in \mathbb{Z} \tag{10}$$

$$s_i + Q^i p_m \geq Q^i P^i \qquad \forall i \in I \quad [x_i] \tag{11}$$

$$s_j + Q^j p_m \geq Q^j P^j - M_j(1 - y_j) \qquad \forall j \in J \quad "[y_j]" \tag{12}$$

$$\sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j \geq \sum_i s_i + \sum_j s_j \tag{13}$$

$$s_i, s_j \geq 0, \quad param.\ M_j >> 0 \tag{14}$$

Third way: yields a useful primal-dual framework

Consider a block order selection $J = J_0 \dot\cup J_1$

Primal:

$$\max_{x_i, y_j} \sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j$$

subject to:

$$
\begin{aligned}
& x_i \leq 1 && \forall i \in I && [s_i] && (15)\\
& y_j \leq 1 && \forall j \in J && [s_j] && (16)\\
& y_{j_0} \leq 0 && \forall j_0 \in J_0 \subseteq J && [d_{0_{j_0}}] && (17)\\
& -y_{j_1} \leq -1 && \forall j_1 \in J_1 \subseteq J && [d_{1_{j_1}}] && (18)\\
& \sum_i Q^i x_i + \sum_j Q^j y_j = 0, && && [p_m] && (19)\\
& && x_i,\ y_j \geq 0 && && (20)
\end{aligned}
$$

$$\textbf{Dual:}\ \min \sum_i s_i + \sum_j s_j - \sum_{j_1 \in J_1} d_{1_{j_1}}$$

subject to:

$$s_i + Q^i p_m \geq Q^i P^i \qquad \forall i \in I \quad [x_i] \tag{21}$$

$$s_{j_0} + d_{0_{j_0}} + Q^{j_0} p_m \geq Q^{j_0} P^{j_0} \qquad \forall j_0 \in J_0 \quad [y_{j_0}] \tag{22}$$

$$s_{j_1} - d_{1_{j_1}} + Q^{j_1} p_m \geq Q^{j_1} P^{j_1} \qquad \forall j_1 \in J_1 \quad [y_{j_1}] \tag{23}$$

$$s_i, s_j, d_{j_0}, d_{j_1}, u_m \geq 0 \tag{24}$$

Complementarity constraints

$$s_i(1 - x_i) = 0 \qquad \forall i \in I \tag{25}$$

$$s_{j_0}(1 - y_{j_0}) = 0 \qquad \forall j \in J \tag{26}$$

$$s_{j_1}(1 - y_{j_1}) = 0 \qquad \forall j \in J \tag{27}$$

$$x_i(s_i + Q^i p_m - Q^i P^i) = 0 \qquad \forall i \in I \tag{28}$$

$$y_{j_0}(s_{j_0} + d_{0_{j_0}} + Q^{j_0} p_m - Q^{j_0} P^{j_0}) = 0 \qquad \forall j \in J_0 \tag{29}$$

$$y_{j_1}(s_{j_1} - d_{1_{j_1}} + Q^{j_1} p_m - Q^{j_1} P^{j_1}) = 0 \qquad \forall j \in J_1 \tag{30}$$

$$y_{j_0} d_{0_{j_0}} = 0, \quad (1 - x_{j_1}) d_{1_{j_1}} = 0 \qquad \forall j_0 \in J_0, \forall j_1 \in J_1 \tag{31}$$

With primal, dual and complementarity constraints:

- d_{0_j} is an *upper bound* on the opportunity cost of order j
- d_{1_j} is an *upper bound* on the actual loss of (executed) order j

Block order selection $J = J_0 \,\dot{\cup}\, J_1$ not known 'ex ante'.

Decide a selection $J = J_0 \,\dot{\cup}\, J_1$ according to some objective:

- Using equality of objective functions to enforce complementarity conditions
- .... and using a 'dispatcher'

Primal-dual framework:

Feasible Set LMM defined by:

$$x_i \leq 1 \qquad \forall i \in I$$

$$y_j \leq 1 \qquad \forall j \in J$$

$$\sum_i Q^i x_i + \sum_j Q^j y_j = 0$$

$$x_i, y_j \geq 0, \quad y_j \in \mathbb{Z}$$

$$s_i + Q^i p_m \geq Q^i P^i \qquad \forall i \in I$$

$$s_j + d_{0_j} - \cancel{d_{1_j}} + Q^j p_m \geq Q^j P^j \qquad \forall j \in J$$

$$d_{0_j} \leq M_j(1 - y_j) \qquad \forall j \in J$$

d_{0_j} *upper bound* on the opportunity cost of order j

$$\cancel{d_{1_j} \leq M_j y_j} \qquad \forall j \in J$$

d_{1_j} *upper bound* on the actual loss of executed order j

$$s_i, s_j, d_{0_j}, d_{1_j} \geq 0, \quad param. : \ M_j >> 0$$

$$\sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j \geq^{\cdots=} \sum_i s_i + \sum_j s_j - \cancel{\sum_{j_1 \in J_1} d_{1_{j_1}}}$$

Strong duality <-> 'relaxed complementarity constraints'

Core European market model, new formulation:

No auxiliary variables at all & *tractable for large-scale instances*

$$\max_{x_i, y_j, p_m, s_i, s_j} \quad \sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j$$

subject to:

$$x_i \leq 1 \qquad \forall i \in I \quad [s_i] \qquad (32)$$

$$y_j \leq 1 \qquad \forall j \in J \quad \text{"}[s_j]\text{"} \qquad (33)$$

$$\sum_i Q^i x_i + \sum_j Q^j y_j = 0 \qquad [p_m] \qquad (34)$$

$$x_i, y_j \geq 0, \quad y_j \in \mathbb{Z} \qquad (35)$$

$$s_i + Q^i p_m \geq Q^i P^i \qquad \forall i \in I \quad [x_i] \qquad (36)$$

$$s_j + Q^j p_m \geq Q^j P^j - M_j(1 - y_j) \qquad \forall j \in J \quad \text{"}[y_j]\text{"} \qquad (37)$$

$$\sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j \geq \sum_i s_i + \sum_j s_j \qquad (38)$$

$$s_i, s_j \geq 0, \quad param.\ M_j >> 0 \qquad (39)$$

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New primal-dual formulation - [b] - M. Van Vyve & M.

- No auxiliary variables at all !
- Could then derive a powerful Benders decomposition with strengthened Benders cuts improving on Martin-Muller-Pokutta [d] (both propositions: projection on the space of primal variables)
- Martin-Muller-Pokutta: Branch-and-Cut with exact (globally valid) no-good cuts:

$$\sum_{j|y_j^*=1}(1-y_j)+\sum_{j|y_j^*=0}y_j \geq 0$$

- with the new stuff: recovering these globally valid cuts + stronger *locally valid* cuts:

$$\sum_{j|y_j^*=1}(1-y_j) \geq 0$$

- needed with piecewise linear bid curves ($\rightarrow$ quad. prog. setting, using convex quad. prog. duality)
- 'State-of-the-art'

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   - Equilibrium with block orders ? : Examples
   - Primal Program (welfare maximizing dispatch)
   - Duality and Linear Equilibrium Prices
2. Get rid of the subset of compl. constr. ? Three ways
   - Linearise each of them...
   - Equality of objective functions + linearisation of quadratic terms
   - New primal-dual framework ...
3. Algorithmic application - Strengthened (locally valid) Benders cuts
4. Minimizing opportunity costs, Maximizing the traded volume, etc
5. Numerical results
   - Maximizing welfare, new approach
   - Opportunity costs vs Welfare
6. Conclusions and Extensions

Primal-dual framework:

Feasible Set LMM defined by:

$$x_i \leq 1 \qquad \forall i \in I$$

$$y_j \leq 1 \qquad \forall j \in J$$

$$\sum_i Q^i x_i + \sum_j Q^j y_j = 0$$

$$x_i, y_j \geq 0, \quad y_j \in \mathbb{Z}$$

$$s_i + Q^i p_m \geq Q^i P^i \qquad \forall i \in I$$

$$s_j + d_{0_j} - \cancel{d_{1_j}} + Q^j p_m \geq Q^j P^j \qquad \forall j \in J$$

$d_{0_j} \leq M_j(1 - y_j)$ $\quad d_{0_j}$ *upper bound* on the opportunity cost of order j $\quad \forall j \in J$

$\cancel{d_{1_j} \leq M_j y_j}$ $\quad d_{1_j}$ *upper bound* on the actual loss of executed order j $\quad \forall j \in J$

$$s_i, s_j, d_{0_j}, d_{1_j} \geq 0, \quad param.: \quad M_j >> 0$$

$$\sum_i Q^i P^i x_i + \sum_j Q^j P^j y_j \geq^{\cdots=} \sum_i s_i + \sum_j s_j - \cancel{\sum_{j_1 \in J_1} d_{1_{j_1}}}$$

Strong duality <-> ‘relaxed complementarity constraints’

Other optimization problems under European rules

$d_{0_j} \geq 0$ *upper bound* on the opportunity cost of block order j...

Minimizing opportunity costs ?

$$\min \sum d_{0_j}$$

Minimizing # PRBs ?

$$\min \sum z_{0_j} \text{ s.t. } Mz_{0_j} \geq d_{0_j} \;\&\; z_{0_j} \in \{0,1\}, \quad \forall j \in J$$

Maximizing the traded volume ?

$$\max \quad \sum_{i|Q^i>0} Q^i x_i + \sum_{j|Q^j>0} Q^j y_j$$

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Numerical results: welfare optimization

- Real data of 2011, thanks to Apx-Endex and Epex Spot
- Belgium, France, Germany and the Nederlands, 24 time slots
- time limit: 10 min., about 60 000 cont. vars, 600/700 bin. vars
- Branch-and-cut in AIMMS, using
  Cplex 12.5 with locally valid lazy constraints callbacks (not in Gurobi)
  platform: windows 7 64, i5 with 4 cores @ 3.10 GHz, 4 GB RAM

Stepwise preference curves (linearisation):

	Solved instances	Running time (solved instances, sec)	Final abs. gap (unsolved instances)	Nodes (solved - unsolved) instances	Cuts (solved - unsolved) instances
New MILP formulation	84%	104.42	418.16	43 - 33584	/
Decomposition Procedure	72.78%	6.47	402.05	16 - 1430	8 - 3492

Quadratic setting:

	Solved instances	Running time (solved instances, sec)	Final abs. gap (unsolved instances)	Nodes (solved - unsolved) instances	Cuts (solved - unsolved) instances
Decomposition Procedure	70.41%	16.70	370.91	11 - 619	7 - 1382

Many blocks (almost binary orders only):

	Solved instances	Running time (solved instances, sec)	Final abs. gap (unsolved instances)	Nodes (solved - unsolved) instances	Cuts (solved - unsolved) instances
New MILP formulation	100%	4.17	/	40797 - /	-
Decomposition Procedure	78%	13.82	9303.16	64564 / 937172	1662 / 82497

Opportunity costs vs Welfare Maximization

CWE Region, some instances from 2011 (see [e] for more details)

# Instances	WMP Solution	OCMP solution		Comparison		
	OC	OC - best solution	OC - best bound	Delta Welfare	Delta OC	Final Gap / Initial Gap
1	961.37	961.37	927.13			4%
2	156.81	156.81	156.81			0%
3						
4	783.19	265.97	127.29	20.00	517.22	18%
5	249.24	132.16	132.16	67.00	117.08	0%
6	5669.17	5669.17	590.11			90%
7	2257.03	2257.03	546.77			76%
8	581.56	581.56	581.56			0%
9	504.24	481.74	145.63	29.00	22.50	67%
10	535.21	203.38	111.59	59.00	331.83	17%

Short conclusion and Extensions

Conclusions / Extensions

- Well-known nowadays: MIP formulation issues really matter ...
- framework useful both for economic modelling (min. opportunity costs, max traded volume, etc) and algorithmically (feeding Cplex or e.g. Benders decomposition to derive new cuts)
- The same approach (whole MIP) extends to complex bids with a Minimum income condition ! (WP available within a couple of days)
- Many other things to say e.g. about network/spatial equilibrium, etc

bibliography

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[c] E.J. Zak, S. Ammari, and K.W. Cheung. Modeling price-based decisions in advanced electricity markets. In European Energy Market (EEM), 2012 9th International Conference on the, May 2012

[d] Alexander Martin, Johannes C. Müller, and Sebastian Pokutta. Strict linear prices in nonconvex european day-ahead electricity markets. Optimization Methods and Software, 2014

[e] M. & Mathieu Van Vyve, Minimizing opportunity costs of paradoxically rejected block orders in European day-ahead electricity markets, European Energy Market (EEM), 2014 (see IEEE Xplore)

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